Examples for Integrals

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$$\int_0^1 sin^{-1}xdx$$

Let $u = sin^{-1}x, dv = 1du$ so $du = \frac{1}{\sqrt{1-x^2}}dx, v = x$ (Apply integration by parts)

$= xsin^{-1}x\}_0^1 - \displaystyle\int_0^1 \frac{1}{\sqrt{1-x^2}}xdx$

Let $u = 1 - x^2$ so $du = -2xdx$

When $x = 0, u = 1$ when $x = 1, u = 0$ (Apply integration by substitution)

$= xsin^{-1}x\}_0^1 - \displaystyle\int_1^0 -\frac{1}{2\sqrt{u}}du$

$= xsin^{-1}x\}_0^1 + \displaystyle\frac{1}{2}\int_1^0 u^{-\frac{1}{2}}du$

$= xsin^{-1}x\}_0^1 + \displaystyle\frac{1}{2}(2\sqrt{u}\}_1^0$

$= xsin^{-1}x\}_0^1 + \sqrt{u}\}_1^0$

$= sin^{-1}(1) - 0 + 0 - 1$

$= \frac{\pi}{2} - 1$

$\int_0^{\frac{\pi}{6}} \tan^2 x \sec^3 xdx$

Let $u = sec^3x, dv = tan^2xdx$ (Apply integration by parts)

$= \sec^3 x(\tan x - x) - 3\int \sec^3 x \tan x(\tan x - x)dx$

$= \sec^3 x(\tan x - x) - 3\int \tan^2 x \sec^3 xdx + 3\int x\sec^3 x \tan xdx$

$4\int \tan^2 x \sec^3 xdx = \sec^3 x(\tan x - x) + 3\int x\sec^3 x \tan xdx$

(For $3\int x\sec^3 x \tan xdx$) Let $u = x, dv = sec^3x \tan xdx$ (Apply integration by parts)

$3\int x\sec^3 x \tan xdx = x\sec^3 x - \int \sec^3 xdx$

(For $\int \sec^3 xdx$) $\int \sec^3 xdx = \int\sqrt{1 + tan^2x}dx$

Let $t = tanx$ (Apply integration by substitution)

$= \frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln\left(t + \sqrt{1+t^2}\right)$

Put all of them together(final answer):

$= \frac{1}{4}\left(\sec^3 x \tan x - \frac{1}{2}\left(\tan^{-1} x\sqrt{1 + (\tan^{-1} x)^2} + \ln\left(\tan^{-1} x + \sqrt{1 + (\tan^{-1} x)^2}\right)\right)\right)\}_0^{\frac{\pi}{6}}$

$= \frac{1}{3} + \frac{2}{9} - \frac{\ln(3)}{4} + \frac{1}{16}(4 + \ln(27))$

$= \frac{5}{36} - \frac{\ln(3)}{16}$

$\int_{256}^{4294967296} \frac{2}{\sqrt{x}-3\sqrt[4]{x}+2} dx$

Let $t = \sqrt[4]{x}$ $dx = 4t^3 dt$ (Apply integration by substitution)

$= \int \frac{8t^3}{t^2-3t+2} dt$

$= \int \frac{8t^3-8t^2+8t^2}{(t-1)(t-2)} dt$

$= \int \frac{8t^2}{t-2} dt + \quad \int \frac{8t^2-8t+8t}{(t-1)(t-2)} dt$ (Apply integration by partial fractions)

$= \int \frac{8t^2}{t-2} dt + \int \frac{8t}{t-2} dt + \int \frac{16(t-1)-8(t-2)}{(t-1)(t-2)} dt$

$= \int \frac{8t^2}{t-2} dt + \int \frac{8t}{t-2} dt + \int \frac{16}{t-2} dt - \int \frac{8}{t-1} dt$

$= \int 8(t-2) dt + \int \frac{56}{t-2} dt \quad + \int 32 dt \quad + \int \frac{8t}{t-2} dt + \int \frac{16}{t-2} dt - \int \frac{8}{t-1} dt$

$= \int 8(t-2) dt + \int \frac{56}{t-2} dt \quad + \int 32 dt \quad + \int \frac{8t}{t-2} dt - \int \frac{8}{t-1} dt$

$= 4(t-2)^2 + 56 \ln|t-2| + 32t + 8 \ln|t-2| + 8t \quad - 8ln|t-1|$

$= 4(t-2)^2 + 64 \ln|t-2| + 40t - 8ln|t-1|$

Put $t = \sqrt[4]{x}$ back into the function:

$= 4(\sqrt[4]{x}-2)^2 + 64 \ln|\sqrt[4]{x}-2| + 40\sqrt[4]{x} - 8ln|\sqrt[4]{x}-1|\}_{256}^{4294967296}$

$= 64\left(4192 + \ln(254)\right) - 8\ln(255) - -8\left(20 + \ln\left(\frac{256}{3}\right)\right)$

$= -8\left(-33516 + \ln(85) - 8\ln(127)\right)$