On demand equation, revenue, cost and profit

Demand equation describes the relationship between the price of an item and the number of units that will sell at that price. It is a basic principal of economics that the demand relationship has the characteristic that an increase in price will lead to a decrease in demand. The simplest such relationship is a linear one. In practice, demand relationships are often non-linear. It is usual to use the variables p for ***the price of a unit***, and q for the ***quantity demanded***. Please do this consistently throughout the term.

Revenue, R, is the amount of money that a company receives by selling the objects at the set price; $R = p.q$.

Cost, C, is the amount a company spends to build a certain number of objects. This is often broken down into fixed costs (i.e. the things the company has to pay for even if it produces 0 units such as rent, wages, maintenance, etc.) and variable costs (things that increase with the number of units made).

Profit is what the company is left with once the product is sold, the revenue is taken in and the costs are paid off; $P(q) = R(q)C(q)$.

In the problem below, we will plot the demand relationship on the (q, p)-plane and treat p as a function of q. However, note that practically q is not the independent variable in these problems: although the producer has the ability to set the price p, the demand relationship is NOT in control of the producer, so setting p determines how many items q will be sold.

Example:

Opple Inc. is the only manufacturer of the popular oPad. Opple estimates that when the price of the oPad is $200, then the weekly demand for it is 5000 units. For every $1 increase in the price, the weekly demand decreases by 50 units. Assume that the fixed costs of production on a weekly basis are $100,000, and the variable costs of production are $75 per unit.

a. Find the linear demand equation for the oPad. Use the notation p for the unit price and q for the weekly demand.

b. Find the weekly cost function, $C = C(q)$, for producing q oPads per week. Note that $C(q)$ is a linear function.

c. Find the weekly revenue function, $R = R(q)$. Note that $R(q)$ is a quadratic function.

d. The break-even points are where Cost equals Revenue; that is, where $C(q) = R(q)$. Find the break-even points for the oPad.

e. On the same set of axes, sketch graphs of $C = C(q)$ and $R = R(q)$ and use these graphs to help you explain why there are two break-even points.

f. *Profit* is de ned as Revenue minus Cost: $P(q) = R(q)$

g. Graph $P = P(q)$ on the same axes as you sketched the graphs of $C(q)$ and $R(q)$. On this graph, indicate the regions of profit $(P(q) > 0)$ and loss $(P(q) < 0)$.