Ch 1 Review Answer key

1ad, 2ad, 3ad, 4ac, 6ac, 7ac, 8, 11ab, 12, 13, 15ac, 16ab, 17a, 18ab, 20all, 21

1. a) NOT arithmetic

b) $d = -3x^2 \quad t_1 = -2x^2$

so $t_n = -2x^2 + (n-1)(-3x^2)$

$t_n = -2x^2 - 3nx^2 + 3x^2$

$\underline{t_n = x^2 - 3nx^2}$

2. a) 1, −3, −7, −11, −15

d) $c+1,\ 2c-1,\ 3c-3,\ 4c-5,\ 5c-7$

3. a) $t_5 = 16 \quad t_8 = 25$

$t_5 = t_1 + (5-1)d \qquad t_8 = t_1 + (8-1)d$

$16 = t_1 + 4d \qquad 25 = t_1 + 7d$

$$\begin{array}{r} 25 = \cancel{t_1} + 7d \\ -(16 = \cancel{t_1} + 4d) \\ \hline 9 = 3d \end{array}$$

$\frac{9}{3} = \frac{3d}{3}$

$d = 3$

so $16 = t_1 + 4(3)$

$-12 \qquad -12$

$t_1 = 4$

$\underline{t_1 = 4 \quad d = 3}$

d) $t_7 = 37$

$t_7 = t_1 + (7-1)d$

$37 = t_1 + 6d$

$t_{10} = 22$

$t_{10} = t_1 + (10-1)d$

$22 = t_1 + 9d$

$$\begin{aligned} 22 &= \cancel{t_1} + 9d \\ -(37 &= \cancel{t_1} + 6d) \\ \hline -15 &= 3d \end{aligned}$$

$$\frac{-15}{3} = \frac{3d}{3}$$

$$\underline{\underline{d = -5}}$$

$22 = t_1 + 9(-5)$

$22 = t_1 - 45$

$+45 \qquad +45$

$$\underline{\underline{67 = t_1}}$$

4. a) $3, 5, 7, \ldots, 129$

$t_n = t_1 + (n-1)d$ where $t_n = 129$, $t_1 = 3$, $d = 2$

so $129 = 3 + (n-1)2$

$129 = 3 + 2n - 2$

$129 = 1 + 2n$

$-1 \qquad -1$

$$\frac{128}{2} = \frac{2n}{2}$$

$$\underline{\underline{n = 64}}$$

c) $t_1 = -29 \quad d = 5 \quad t_n = 126$

$126 = -29 + (n-1)(5)$

$126 = -29 + 5n - 5$

$126 = 5n - 34$

$+34 \qquad +34$

$$\frac{160}{5} = \frac{5n}{5} \Rightarrow n = 32$$

6. a) $S_{10} = \frac{10}{2}[2(2) + (10-1)(6)]$

$= 5[4 + 54] = \underline{\underline{290}}$

c) $S_{10} = \frac{10}{2}[2(45) + (10-1)(-6)]$

$= 5[90 - 54] = \underline{\underline{180}}$

7. a) Find n first:

$t_n = t_1 + (n-1)(d)$

$92 = 2 + (n-1)(5)$

$92 = 2 + 5n - 5$

$92 = 5n - 3$

$+3 \qquad +3$

$\frac{95}{5} = \frac{5n}{5}$

$n = 19$

$S_{19} = \frac{19}{2}[2 + 92]$

$= \underline{\underline{893}}$

c) Find n first:

$-70 = 20 + (n-1)(-6)$

$-70 = 20 - 6n + 6$

$-70 = 26 - 6n$

$-26 \qquad -26$

$\frac{-96}{-6} = \frac{-6n}{-6}$

$n = 16$

$S_{16} = \frac{16}{2}[20 + (-70)]$

$= 8[-50]$

$= \underline{\underline{-400}}$

8. $S_{10} = 210$ $S_{20} = 820$

$210 = \frac{10}{2}[2t_1 + 9d]$ $820 = \frac{20}{2}[2t_1 + 19d]$

$210 = 5[2t_1 + 9d]$ $820 = 10[2t_1 + 19d]$

$210 = 10t_1 + 45d$ $820 = 20t_1 + 190d$

$(\div 10)\ 820 = 20t_1 + 190d \Rightarrow 82 = 2t_1 + 19d$

$(\div 5)\ 210 = 10t_1 + 45d \Rightarrow 42 = 2t_1 + 9d$

$\frac{40}{10} = \frac{10d}{10}$

$d = 4$

To find t_1:

$210 = 10t_1 + 45(4)$

$210 = 10t_1 + 180$

$-180 \qquad -180$

$30 = 10t$

$t_1 = 3$

So the first 5 terms are:

3, 7, 11, 15, 19

or 3 + 7 + 11 + 15 + 19

11 a) $t_1 = 3$ $r = 2$ $t_n = 1536$

$\frac{1536}{3} = \frac{3 \cdot (2^{n-1})}{3}$

$512 = 2^{n-1}$

$\Downarrow$

$2^9 = 2^{n-1}$ $\therefore$ $\underline{\underline{n = 10}}$

b) $t_1 = -409.6 = \frac{-2048}{5}$

$r = -.25 = -\frac{1}{4}$

$t_n = .025 = \frac{1}{40}$

so: $\frac{1}{40} = \frac{-2048}{5}\left(-\frac{1}{4}\right)^{n-1}$ (x both sides by 5, ÷ both sides by -2048)

$\frac{\cancel{5}^{1}}{-2048} \cdot \frac{1}{\cancel{40}_8} = \left(-\frac{1}{4}\right)^{n-1}$

$-\frac{1}{16384} = \left(-\frac{1}{4}\right)^{n-1}$ $\left(-16384 = (-4)^7\right)$

$\left(-\frac{1}{4}\right)^7 = \left(-\frac{1}{4}\right)^{n-1}$ $\therefore n = 8$

12. $t_1 = 2$ $t_5 = 162$

$\frac{162}{2} = \frac{(2)}{2} r^4$

$81 = r^4$

$\downarrow$

$3^4 = r^4$ $\therefore \underline{\underline{r = 3}}$ (or $r = -3$).

so $2, 6, 18, 54, 162.$

13. a) $t_3 = 36$ and $t_4 = 108$

Method I: $\frac{t_4}{t_3} = r$ so $\frac{108}{36} = 3 = r$

$t_4 = t_1 r^3$

$108 = t_1 \cdot 3^3$

$\frac{108}{27} = \frac{t_1 \cdot 27}{27}$ $t_1 = 4$

General term: $\underline{\underline{t_n = 4 \cdot 3^{n-1}}}$

<u>OR</u> Method II

$t_3 = t_1 \cdot r^2$ $t_4 = t_1 \cdot r^3$

$36 = t_1 \cdot r^2$ $108 = t_1 \cdot r^3$

$\frac{108}{36} = \frac{\cancel{t_1} \cdot r^3}{\cancel{t_1} r^2}$

$3 = r$ $\frac{36}{3^2} = \frac{t_1 (3)^2}{3^2}$

$t_1 = 4$

so $t_n = \underline{\underline{4 \cdot 3^{n-1}}}$

13 b. $t_3 = 99$ $t_5 = 11$

$99 = t_1 \cdot r^2$ $11 = t_1 \cdot r^4$

$$\frac{11}{99} = \frac{\cancel{t_1} \cdot r^4}{\cancel{t_1} r^2}$$

$\frac{1}{9} = r^2$ so $r = \frac{1}{3}$

$$99 = t_1 \left(\frac{1}{3}\right)^2$$

$$9 \cdot 99 = t_1 \cdot \frac{1}{9} \cdot 9$$

$$t_1 = \underline{891}$$

So $t_n = 891 \cdot \left(\frac{1}{3}\right)^{n-1}$

15 a) $t_1 = 24$ $r = -\frac{1}{2}$ $n = 10$

$$S_{10} = \frac{24((-1/2)^{10} - 1)}{(-1/2) - 1} = \frac{1023}{64}$$

c) $r = -1$ $t_1 = 8$ $n = 40$

$$S_{40} = \frac{8((-1)^{40} - 1)}{-1 - 1} = 0$$

16 a) $S_9 = \frac{6(2^9 - 1)}{2 - 1} = 3066$

b) $S_8 = \frac{\frac{1}{2}(4^8 - 1)}{4 - 1} = 10922.5$

17. a) $S_n = \dfrac{(.5)(15) - 960}{.5 - 1} = \underline{\underline{1905}}$

18. a) $t_1 = 7971615 \quad r = \dfrac{2}{3}$

$t_n = 92160 = 7971615\left(\dfrac{2}{3}\right)^{n-1}$

$\dfrac{92160 \div 5}{7971615 \div 5} = \left(\dfrac{2}{3}\right)^{n-1}$

$\Downarrow$

$\dfrac{18432 \div 3}{1594323 \div 3}$

$\Downarrow$

$\dfrac{6144 \div 3}{531441 \div 3}$

$\Downarrow$

$\dfrac{2048}{177147} = \dfrac{2^{11}}{3^{11}} = \left(\dfrac{2}{3}\right)^{11} = \left(\dfrac{2}{3}\right)^{n-1}$

$\therefore \underline{\underline{n = 12}}$

18. b) $t_1 = 1 \quad r = 3x^2$

$t_n = \underbrace{243x^{10}} = (1)(3x^2)^{n-1}$

$\downarrow$

$(3x^2)^5 = (3x^2)^{n-1} \quad \therefore \underline{\underline{n = 6}}$

20\. a) $r = -\frac{1}{4}$ ∴ convergent & finite sum exists

$$S_\infty = \frac{-6}{1-(1/4)} = \frac{-256}{5} \text{ or } -51.2$$

b) $r = 2$ ∴ divergent and no finite sum exists.

c) $r = .2$ ∴ convergent

$$S_\infty = \frac{6.1}{1-.2} = 7.625 \text{ or } \frac{61}{8}$$

d). $r = -2.5$ ∴ divergent; sum does not exist.

21\. $S_\infty = 120$, $r = -\frac{2}{5}$

$$120 = \frac{t_1}{1-(\frac{-2}{5})}$$

$$\frac{7}{5} \cdot 120 = \frac{t_1}{\cancel{7/5}} \cdot \cancel{7/5}$$

$$t_1 = \underline{\underline{168}}$$

so: $168 - \frac{336}{5} + \frac{672}{25} - \ldots$

Chapter 1 Review,

1. a) not arithmetic
   b) arithmetic; $d = \frac{1}{2}$, $t_n = 1\frac{1}{2} + \frac{1}{2}n$
   c) not arithmetic
   d) arithmetic; $d = -3x^2$, $t_n = x^2 - 3nx^2$
2. a) 1, −3, −7, −11, −15
   b) −6, 0, 12, 18, 24
   c) $5m$, $5m + 3$, $5m + 6$, $5m + 9$, $5m + 12$
   d) $c + 1$, $2c - 1$, $3c - 3$, $4c - 5$, $5c - 7$
3. a) $t_1 = 4, d = 3$ b) $t_1 = 42, d = 2$
   c) $t_1 = -19, d = 7$ d) $t_1 = 67, d = -5$
4. a) 64 b) 56
   c) 32 d) 28
5. $30 000
6. a) 290 b) 600
   c) 180 d) 375
7. a) 893 b) 3604
   c) −400 d) 0
8. 3 + 7 + 11 + 15 + 19
9. $n = 21$
10. a) 1 and 25 or −1 and −25 b) 15 and 75
11. a) $n = 10$ b) $n = 8$
    c) $n = 11$ d) $n = 8$
12. $r = \pm 3$; $\pm 6, 18, \pm 54$
13. a) $t_n = 4(3)^{n-1}$ b) $t_n = 891\left(\frac{1}{3}\right)^{n-1}$
14. 6 reductions
15. a) $t_1 = 24, r = -\frac{1}{2}, n = 10; S_{10} = \frac{1023}{64}$
    b) $t_1 = 0.3, r = \frac{1}{100}, n = 15; S_{15} = \frac{10}{33}$
    c) $t_1 = 8, r = -1, n = 40; S_{40} = 0$
    d) $t_1 = 1, r = -\frac{1}{3}, n = 12;\ S_{12} = \frac{265\,721}{354\,294}$
16. a) $S_n = 3066$ b) $S_n = 10\,922.5$
17. a) $S_n = 1905$ b) $S_n = -250\,954$
18. a) 12 terms b) 6 terms
19. 11 weeks
20. a) convergent; $S_\infty = -\frac{256}{5}$
    b) divergent; sum does not exist
    c) convergent; $S_\infty = \frac{61}{8}$
    d) divergent; sum does not exist
21. $t_1 = 168$; $168 - \frac{336}{5} + \frac{672}{25} - \cdots$
22. 300 cm