2.9 Chapter Review

#1a) $f(x) = 1 - 2x + 3x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(1 - 2(x+h) + 3(x+h)^2) - (1 - 2x + 3x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} - \cancel{2x} - 2h + \cancel{3x^2} + 6xh + 3h^2 - \cancel{1} + \cancel{2x} - \cancel{3x^2}}{h}$$

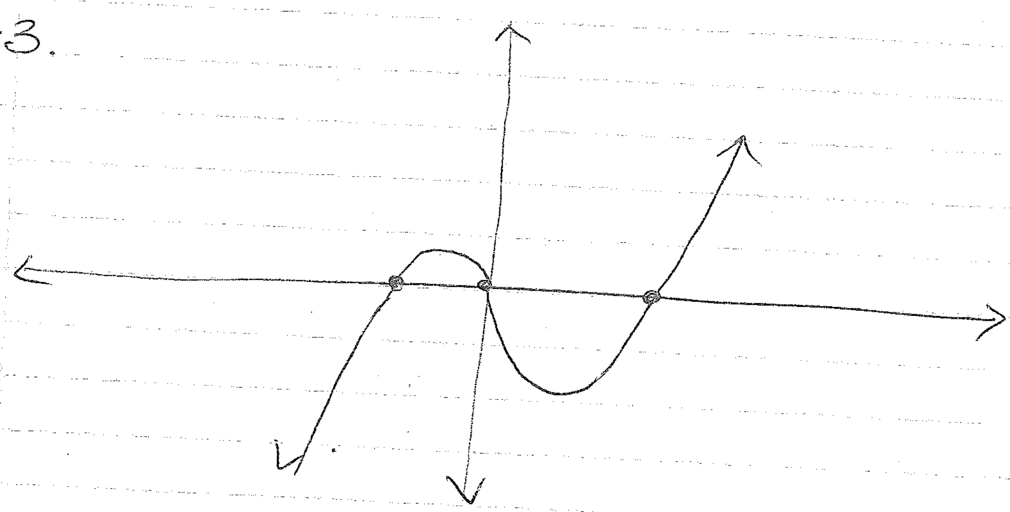
$$= \lim_{h \to 0} \frac{3h^2 + 6xh - 2h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3h + 6x - 2)}{\cancel{h}}$$

$$= 3(0) + 6x - 2$$

$$= \underline{\underline{6x - 2}}$$

#3.



4. a) $y' = 36x^2 + 8$ b) $(2\pi+2)(x^{\pi}) = y'$

c) $y' = 2 + \frac{3}{x^2}$ d) $y' = \frac{6}{5}x^{1/5}$

e) $y' = \sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right) + \frac{1}{2\sqrt{x}}(5-\sqrt{x})$

$$= \frac{-\sqrt{x} + 5 - \sqrt{x}}{2\sqrt{x}}$$

$$= \boxed{\frac{5-2\sqrt{x}}{2\sqrt{x}}} = \frac{5}{2\sqrt{x}} - \frac{2\sqrt{x}}{2\sqrt{x}} = \boxed{\frac{5}{2\sqrt{x}} - 1}$$

OR

f) $y' = \frac{\sqrt{x}(2x-2) - \frac{1}{2}x^{-1/2}(x^2-2x)}{x}$

$$= \frac{2x^{3/2} - 2x^{1/2} - \frac{1}{2}x^{3/2} + x^{1/2}}{x}$$

$$= \frac{\cancel{x}\left[2x^{1/2} - 2x^{-1/2} - \frac{1}{2}x^{1/2} + x^{-1/2}\right]}{\cancel{x}}$$

$$= \frac{2\sqrt{x}}{2\sqrt{x}}2\sqrt{x} - \frac{2 \cdot 2}{2\sqrt{x}} - \frac{\sqrt{x}\sqrt{x}}{\sqrt{x}\,2} + \frac{2 \cdot 1}{2\sqrt{x}}$$

$$= \frac{4x - 4 - x + 2}{2\sqrt{x}}$$

$$= \frac{3x-2}{2\sqrt{x}}$$

g) $$y' = \frac{(1+3x)\overset{\text{oops}}{(\cancel{2x-1})}(2) - (2x-1)(3)}{(1+3x)^2}$$

$$= \frac{2+6x-6x+3}{(1+3x)^2}$$

$$= \frac{5}{(1+3x)^2}$$

h) $$y' = 7(2x^3-1)^6(6x^2)$$

$$= 42x^2(2x^3-1)^6$$

i) $$y' = (x^2+x)\left(\tfrac{1}{\cancel{2}}(1-x^2)^{-1/2} \cdot (-\cancel{2}x)\right) + (2x+1)(1-x^2)^{1/2}$$

$$= -\frac{x^3+x^2}{(1-x^2)^{1/2}} + (2x+1)(1-x^2)^{1/2}\,\frac{(1-x^2)^{1/2}}{(1-x^2)^{1/2}}$$

$2x-2x^3+1-x^2$

$$= \frac{-x^3-x^2+(2x+1)(1-x^2)}{(1-x^2)^{1/2}}$$

$$= \frac{-3x^3-2x^2+2x+1}{\sqrt{1-x^2}}$$

j) $$y' = \frac{(2-x)(6x)-(-1)(3x^2+1)}{(2-x)^2}$$

$$= \frac{12x-6x^2+3x^2+1}{(2-x)^2}$$

$$= \frac{1+12x-3x^2}{(2-x)^2}$$

k) $h(x) = (2x^4-1)^{-1/3}$

$$h' = -\frac{1}{3}(2x^4-1)^{-4/3}(8x^3)$$

$$= \frac{-8x^3}{3\sqrt[3]{(2x^4-1)^4}}$$

l) $$F'(x) = (x^4+1)^3(-2) + (1-2x)(3)(x^4+1)^2(4x^3)$$

$$= -2(x^4+1)^3 + 12x^3(1-2x)(x^4+1)^2$$

$$= -2(x^4+1)^2\left[x^4+1-6x^3\overset{(1-2x)}{\cancel{(x^4+1-12x^8-12x}}\right]$$

$$= -2(x^4+1)^2\left[x^4+1-6x^3+12x^4\right]$$

$$= -2(x^4+1)^2\left[13x^4-6x^3+1\right]$$

m) $$f(t) = \frac{t}{(1+2t)^{1/2}}$$

$$f'(t) = \frac{(1+2t)^{1/2}(1) - \frac{1}{2}(1+2t)^{-1/2}(t)(2)}{(1+2t)}$$

$$= \frac{(1+2t)^{1/2} + \frac{(-t)}{(1+2t)^{1/2}}}{(1+2t)} = \frac{\frac{1+2t-t}{(1+2t)^{1/2}}}{(1+2t)}$$

$$= \frac{1+\cancel{3}t}{(1+2t)^{3/2}}$$

n) $g'(t) = 4\left(\frac{t+1}{t+2}\right)^3 \left[\frac{(t+2)(1)-(1)(t+1)}{(t+2)^2}\right]$

$= 4\frac{(t+1)^3}{(t+2)^3}\left[\frac{1}{(t+2)^2}\right]$

$= \boxed{\frac{4(t+1)^3}{(t+2)^5}}$

o) $R(u) = \sqrt[4]{u+1} - \frac{2}{u^2}$

$R'(u) = \frac{1}{4}(u+1)^{-3/4} - (-4u^{-3})$

$= \boxed{\frac{1}{4\sqrt[4]{(u+1)^3}} + \frac{4}{u^3}}$

p) $S(v) = \sqrt{v-(v^2-8)^5}$

$S' = \frac{1}{2}(v-(v^2-8)^5)^{-1/2}\left[1-5(v^2-8)^4(2v)\right]$

$= \frac{1-10v(v^2-8)^4}{2(v-(v^2-8)^5)^{1/2}}$

q) $M(z) = \left(\frac{1+z}{1+z^2}\right)^{1/2}$

$$M' = \frac{1}{2}\left(\frac{1+z}{1+z^2}\right)^{-1/2}\left[\frac{(1+z^2)(1) - (2z)(1+z)}{(1+z^2)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+z}{1+z^2}\right)^{-1/2}\left[\frac{1+z^2 - 2z - 2z^2}{(1+z^2)^2}\right]$$

$$= \frac{1-2z-z^2}{2(1+z)^{1/2}(1+z^2)^{3/2}} \quad \text{OR} \quad \frac{1-2z-z^2}{2\sqrt{1+z}\left(\sqrt{(1+z^2)^3}\right)}$$

r) $F(y) = \frac{1}{2+3y^{-1}}$

$$= (2+3y^{-1})^{-1}$$

$$F'(y) = -(2+3y^{-1})^{-2}(-3y^{-2})$$

$$= \frac{3}{y^2\left(2+\frac{3}{y}\right)^2} = \frac{3}{\left(y\left(2+\frac{3}{y}\right)\right)^2}$$

$$= \frac{3}{(2y+3)^2}$$

#6. Find $\left.\frac{dy}{dx}\right]_{x=1}$ if $y = u^2 - u^3 + 2u^4$

and $u = \frac{x}{2x-1}$

$$\frac{du}{dx} = \frac{(2x-1)(1) - (2)(x)}{(2x-1)^2}$$

$$= \frac{-1}{(2x-1)^2} \quad \text{or} \quad -(2x-1)^{-2}$$

so $\frac{dy}{dx}$ of $y = u^2 - u^3 + 2u^4$

$$y' = 2u\frac{du}{dx} - 3u^2\frac{du}{dx} + 8u^3\left(\frac{du}{dx}\right)$$

$$y' = 2\left(\frac{x}{2x-1}\right)\left(\frac{-1}{(2x-1)^2}\right) - 3\left(\frac{x}{2x-1}\right)^2\frac{-1}{(2x-1)^2} + 8\left(\frac{x}{2x-1}\right)^3 \cdot \frac{-1}{(2x-1)^2}$$

$$= \frac{-2x}{(2x-1)^3} + \frac{3x^2}{(2x-1)^4} - \frac{8x^3}{(2x-1)^5}$$

$$y'(1) = \frac{-2(1)}{1^3} + \frac{3(1)^2}{1^4} - \frac{8(1)^3}{1^5}$$

$$= -2 + 3 - 8$$

$$= \underline{\underline{-7}}$$