1

Chapter 5 Trig Review
Don't lose your identity doing identities!

Name: Key

Solve: $2\cos^2 x - \cos x - 1 = 0$, $0 \le x < 2\pi$

A. $x = 0, \frac{5\pi}{6}, \frac{7\pi}{6}$

B. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

C. $x = \frac{x}{6}, \pi, \frac{11\pi}{6}$

D. $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = 1$

$\frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $x = 0$

Solve: $\csc x = 2$, $0 \le x < 2\pi$

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

B. $x = \frac{\pi}{6}, \frac{11\pi}{6}$

C. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

D. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

$\sin x = \frac{1}{2}$

$\frac{\pi}{6}, \frac{5\pi}{6}$

Solve: $2\sin x = \cos 3x$, where $0 \le x < 2\pi$

A. 0.31, 3.45
B. 2.83, 5.98
C. 0.39, 2.75, 4.03, 5.30
D. 0.98, 2.16, 3.55, 5.89

$x = .3071$
or 3.45

Determine the number of solutions in the interval $0 \le x < 2\pi$ for:

$$\sin ax = \frac{1}{3}, \quad a \text{ is an integer, where } a \ge 1$$

A. 2

B. $\frac{a}{2}$

C. a

D. $2a$

$\sin x = \frac{1}{3}$ has 2 solutions

so $\sin ax = \frac{1}{3}$ has $2a$ sol^{n}s

Solve: $\sin 2x = \frac{1}{\sqrt{2}}$, where $0 \le x < 2\pi$

A. $x = \frac{\pi}{8}, \frac{3\pi}{8}$

B. $x = \frac{\pi}{4}, \frac{3\pi}{4}$

C. $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

D. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sin x = \frac{1}{\sqrt{2}}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$

so $\sin 2x = \frac{1}{\sqrt{2}}$

has:

$\frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{11\pi}{8}$

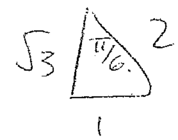
Determine the general solution: $\sin 2x = -\frac{1}{2}$

A. $\frac{7\pi}{12} + 2n\pi$, $\frac{11\pi}{12} + 2n\pi$, n is an integer

B. $\frac{7\pi}{12} + n\pi$, $\frac{11\pi}{12} + n\pi$, n is an integer ✓

C. $\frac{13\pi}{12} + 2n\pi$, $\frac{21\pi}{12} + 2n\pi$, n is an integer

D. $\frac{13\pi}{12} + n\pi$, $\frac{21\pi}{12} + n\pi$, n is an integer



$\sin x = -\frac{1}{2}$

Gives

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

So $\sin 2x = -\frac{1}{2}$

$\frac{7\pi}{12}$ + period

$\frac{11\pi}{12}$ + period.

Determine the restriction(s) for the expression $\frac{\tan\theta}{2\cos\theta - 1}$.

A. $\cos\theta \neq \frac{1}{2}$

B. $\sin\theta \neq 0$

C. $\sin\theta \neq 0$, $\cos\theta \neq \frac{1}{2}$

D. $\cos\theta \neq 0$, $\cos\theta \neq \frac{1}{2}$ ✓

$\cos\theta \neq \frac{1}{2}$.

$\cos\theta \neq 0$.

8. Determine an expression equivalent to $\tan^2\theta\csc\theta+\dfrac{1}{\sin\theta}$.

A. $\sec^3\theta$

B. $\csc^3\theta$

C. $\csc^2\theta\sec\theta$

D. $\sec^2\theta\csc\theta$

$(\sec^2\theta-1)\csc\theta + \csc\theta$

$\csc\theta(\sec^2\theta-1+1)$

9. Determine an expression equivalent to $\dfrac{\tan\theta\csc^2\theta}{\sec^2\theta}$.

A. $\tan\theta$

B. $\cot\theta$

C. $\tan^2\theta$

D. $\tan^3\theta$

$\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{1}{\sin^2\theta}$ $=$ $\dfrac{\cos^2\theta}{\cos\theta\sin\theta}$

$\dfrac{1}{\cos^2\theta}$

10 Determine an expression equivalent to $\cos(\pi+2A)$.

A. $-\cos 2A$

B. $\cos 2A$

C. $-\sin 2A$

D. $\sin 2A$

$\cos\pi\cos 2A - \cancel{\sin\pi}\sin 2A$ 0

$-\cos 2A$

11. Simplify: $\cos 2x\cos x+\sin 2x\sin x$

A. $\cos x$

B. $\sin x$

C. $\cos 3x$

D. $\sin 3x$

$+\ 2\sin x\cos x\sin x$

$(1-2\sin^2 x)\cos x + 2\sin^2 x\cos x$

$\cos x(1-2\sin^2 x+2\sin^2 x)$

$\cos x$

12. Simplify: $\frac{2\sin\theta}{\sin 2\theta}$

A. 1

B. $\cos\theta$

C. $\csc\theta$

D. $\sec\theta$

$\frac{\cancel{2}\cancel{\sin\theta}}{\cancel{2}\cancel{\sin\theta}\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$

Solve algebraically, giving exact values, where $0 \le x < 2\pi$.

$$\sin x = \cos 2x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

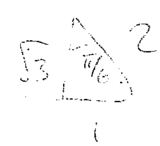
$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$$

Solve algebraically, giving exact values, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$:

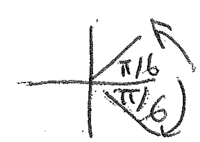
$$2\tan x \cos x - \sqrt{3}\tan x = 0$$



$$\tan x(2\cos x - \sqrt{3}) = 0$$

$$\tan x = 0 \quad \text{or} \quad \cos x = \frac{\sqrt{3}}{2}$$

$$x = 0 \qquad x = \frac{\pi}{6}, \cancel{\frac{11\pi}{6}}$$



$$-\frac{\pi}{6}$$

$$x = -\frac{\pi}{6}, 0, \frac{\pi}{6}$$

Prove the identity:

$$\frac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos x + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$	$\cos x \cdot \dfrac{\cos x}{\sin x}$
$\dfrac{\dfrac{\cos x(1+\cancel{\sin x})}{\sin x}}{\dfrac{(1+\cancel{\sin x})}{\cos x}}$	$\dfrac{\cos^2 x}{\sin x}$
$\dfrac{\cos^2 x}{\sin x}$	

LS = RS ✓

Prove the identity:

$$\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$$

LEFT SIDE

RIGHT SIDE

$$\frac{\cancel{2}\,\cancel{\cos x}(1+\cos x)}{\cancel{2}\sin x\,\cancel{\cos x}}$$

$$\frac{(1+\cos x)(1-\cos x)}{\sin x\,(1-\cos x)}$$

$$\frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

$$\frac{\sin^{\cancel{2}} x}{\cancel{\sin x}\,(1-\cos x)}$$

$$\frac{\sin x}{1-\cos x}$$

LS = RS

✓

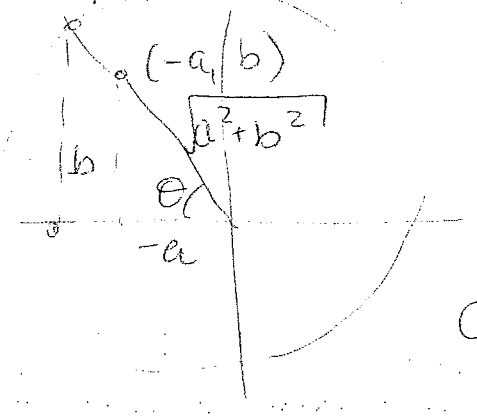
Prove the identity:

$$\frac{\tan x + \sin x}{1 + \cos x} = \frac{1}{\csc 2x} - \frac{\tan x}{\sec 2x}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\sin x \cos x}{\cos x}}{1 + \cos x}$	$\sin 2x - \cos 2x \cdot \tan x$
	$2\sin x \cos x - (2\cos^2 x - 1)\tan x$
	$2\sin x \cos x - 2\cos^{\cancel{2}} x \cdot \dfrac{\sin x}{\cancel{\cos x}} + \tan x$
$\dfrac{\dfrac{\sin x(\cancel{1 + \cos x})}{\cos x}}{(\cancel{1 + \cos x})}$	$+\tan x$
$\tan x$	

LS = RS

9. Point M $(-a,\ b)$ is in quadrant II and lies on the terminal arm of angle θ in standard position. Point N is the point of intersection of the terminal arm of angle θ and the unit circle centred at $(0,\ 0)$. Determine the x-coordinate of point N in terms of a and b.



A. $\dfrac{-a}{\sqrt{a^2+b^2}}$

B. $\dfrac{-b}{\sqrt{a^2+b^2}}$

C. $\dfrac{a}{\sqrt{a^2+b^2}}$

D. $\dfrac{b}{\sqrt{a^2+b^2}}$

$\cos\theta = \dfrac{-a}{\sqrt{a^2+b^2}}$

$\cos\theta = \dfrac{x}{1}$

10. Determine the amplitude of $y = -3\cos 4x + 2$.

A. -4
B. -3
C. 3
D. 4

11. Determine the period of $y = \sin\frac{2\pi}{3}(x-6)$.

A. 3

B. 6

C. $\dfrac{2\pi}{3}$

D. $\dfrac{4\pi}{3}$

$\dfrac{2\pi}{\frac{2\pi}{3}} = \cancel{2\pi} \times \dfrac{3}{\cancel{2\pi}}$

12. Determine the range of the function $y = 6\cos\frac{1}{2}(x-3)+4$.

A. $-6 \le y \le 6$

B. $1 \le y \le 7$

C. $-4 \le y \le 4$

✓ D. $-2 \le y \le 10$

sin.

13. Which of the following lines is an asymptote for the graph of $y = \csc 2x$?

A. $x = 1$

period $= \frac{\pi}{2}$.

B. $x = \frac{\pi}{4}$

✓ C. $x = \frac{\pi}{2}$

D. $x = \frac{3\pi}{4}$

0 $\pi/2$ 2π.

14. State the phase shift of the function $y = -\cos\left(4x - \frac{\pi}{2}\right)$.

✓ A. $\frac{\pi}{8}$ to the right

$-\cos 4\left(x - \frac{\pi}{8}\right)$.

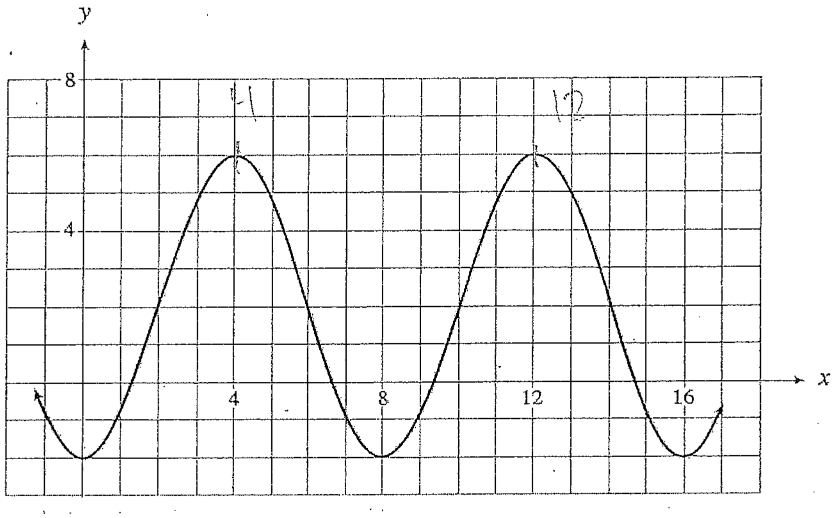
B. $\frac{\pi}{8}$ to the left

$\frac{\pi}{8}$ RIGHT.

C. $\frac{\pi}{2}$ to the right

D. $\frac{\pi}{2}$ to the left

15. If the graph of the function shown below has the equation $y = a\sin b(x-c)+d$, determine the value of b. $(b > 0)$



A. 4

B. 8

C. $\frac{\pi}{4}$

D. $\frac{\pi}{8}$

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{8} = \frac{\pi}{4}.$

period.

$\frac{\pi}{2}$

16. Determine the domain of $f(x) = \tan 2x$.

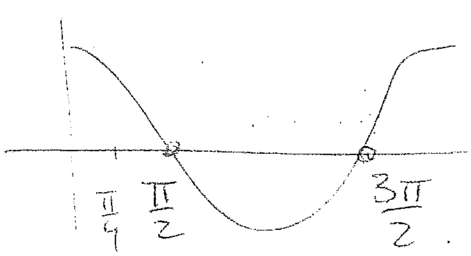
A. $x =$ all real numbers

B. $x =$ all real numbers, $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$, n is an integer

C. $x =$ all real numbers, $x \neq \frac{\pi}{2} + n\pi$, n is an integer

D. $x =$ all real numbers, $x \neq \pi + 2n\pi$, n is an integer

tan is undefined when cos = 0.



$x \neq \frac{\pi}{4} + \frac{n\pi}{2}$

+ nperiod.

$\frac{\pi}{4}$ $\frac{3\pi}{4}$

Calc in rad

17. At a seaport, the depth of the water, d, in metres, at time t hours, during a certain day is given by:

$$d = 3.4 \sin 2\pi \frac{(t - 7.00)}{10.6} + 4.8$$

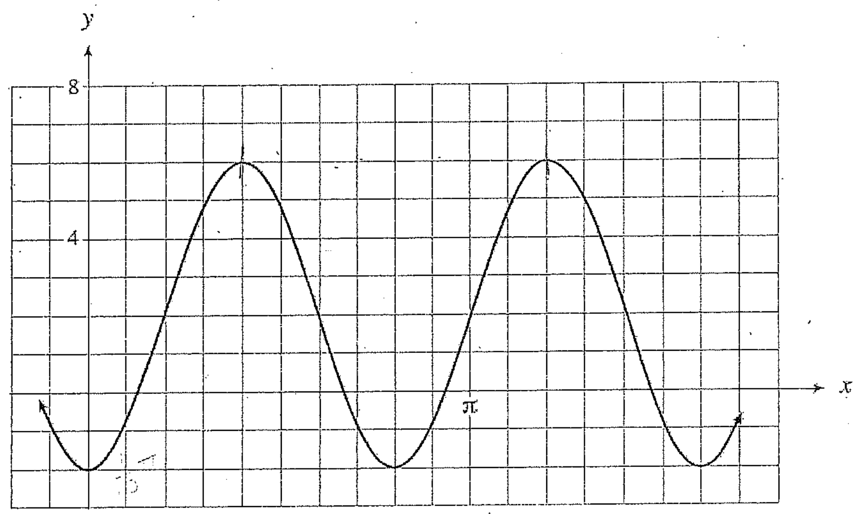
$t = 18.5$

24 hr clock.

On that day, determine the depth of the water at 6:30 p.m.

A. 3.43 m
B. 3.81 m
C. 4.80 m
D. 6.53 m ✓

18. If the graph of the function shown below has the equation $y = a \sin b(x - c) + d$, determine the value of b. $(b > 0)$



$\frac{4\pi}{5}$

A. $\frac{5}{4}$

B. $\frac{5}{2}$ ✓

C. $\frac{2\pi}{5}$

D. $\frac{4\pi}{5}$

$b = \frac{2\pi}{\text{period}} = \frac{2\pi}{\frac{4\pi}{5}} = 2\pi \cdot \frac{5}{4\pi} =$

19. A wheel with radius 20 cm has its centre 30 cm above the ground. It rotates once every 15 seconds. Determine an equation for the height, h, above the ground of a point on the wheel at time, t seconds if this point has a maximum height at $t = 2$ seconds.

A. $h = 20\cos\frac{2\pi}{15}(t+2)+30$

B. $h = 20\cos\frac{2\pi}{15}(t-2)+30$

C. $h = 30\cos\frac{2\pi}{15}(t+2)+20$

D. $h = 30\cos\frac{2\pi}{15}(t-2)+20$

amp = 20

middle : y = 30

T = 15s

$\therefore b = \frac{2\pi}{15}$.

20. A Ferris wheel with a diameter of 60 m rotates once every 48 seconds. At time $t = 0$, a rider is at his lowest height which is 2 m above the ground.

a) Determine a sinusoidal equation that gives the height, h, of the rider above the ground as a function of the elapsed time, t, where h is in metres and t is seconds.

b) Determine the time t when the rider will be 38 m above the ground for the first time after $t = 0$.

$r = 30$

$T = 48$

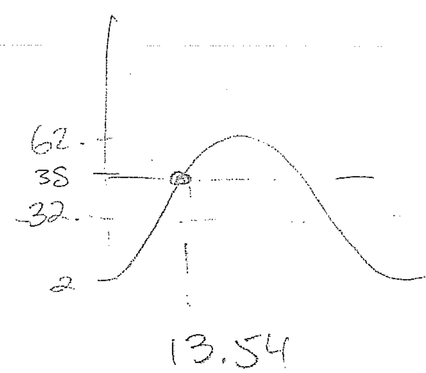
$t = 0$ lowest at 2m above ground

$$b = \frac{2\pi}{48} = \frac{\pi}{24}$$

(a) $$y = -30\cos\left(\frac{\pi}{24}t\right) + 32$$

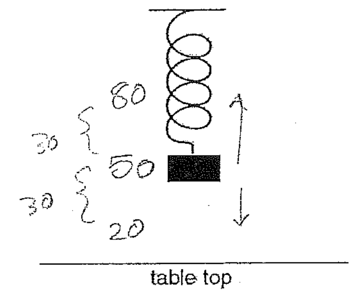
or $$y = 30\sin\frac{\pi}{24}(t-12) + 32$$

(b) at 13.54 s



21. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height h, in cm, above the table top, is approximated by a sinusoidal function of the elapsed time t, in seconds, for a short period of time.

Starts @ a min.



Determine an equation for a sinusoidal function that gives h as a function of t.

$$y = a\cos b(x+c) + d$$

$d = 50\text{ cm}$

$a = 30\text{ cm}$

half period $= .8$

$\therefore$ period $= 1.6$ $\quad b = \frac{2\pi}{1.6}$ or $\frac{20\pi}{16} = \frac{5\pi}{4}$

$$y = -30\cos\left(\frac{5\pi}{4}x\right) + 50$$

OR

$$y = -30\cos\left(\frac{2\pi}{1.6}x\right) + 50$$