Combinations

$_nP_r$ $_nC_r$

Ex 1 a) If 5 sprinters compete in a race, how many different ways can they place 1st, 2nd & 3rd?

S K A J B

S K A

K S A

ORDER MATTERS

$_nP_r$ or $_5P_3 = \frac{5!}{2!} = 60$

This called a PERMUTATION of 5 objects taken 3 at a time.

b) The fastest 3 qualify for the relay team. How many relay teams can be formed?

SKA
KSA > Same team so order does NOT matter

Y Y Y N N

# of ways: $\frac{5!}{3!2!} = 10$

This is called a COMBINATION of 5 objects taken 3 at a time. (unordered or order doesn't matter)

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!}$$

(n ↑, r ↓)

Examples

(1) How many different committees of 3 can be formed from 7 people?
(order has no significance so nCr)

$${}_7C_3 = \frac{7!}{4!3!} = \underline{\underline{35}}$$

(2) Same group, but now the committee must have a president, secretary & treasurer? (now order matters)

nPr $\quad {}_7P_3 = \frac{7!}{4!} = \underline{\underline{210}}$

(3) 7 people, 3 boys + 4 girls
must have 1 boy + 2 girls?

$${}_3C_1 \cdot {}_4C_2 = 3 \cdot 6 = \underline{\underline{18}}$$

(4) Same group (3B, 4G) now need a committee with at least one male.

1B2G OR 2B1G or 3B

$${}_3C_1 \cdot {}_4C_2 + {}_3C_2 \cdot {}_4C_1 + {}_3C_3$$

$$18 + 12 + 1 = \underline{\underline{31}}$$

In a deck of Cards:

- 52 cards in total
- 4 suits (2 Red, 2 black) ♡ ♢ ♤ ♧
- A, 2, 3, ... 10, J, Q, K

Facecards (so 12 of them)

$$P(A) = \frac{\#\text{ of favorable outcomes}}{\text{total \# of possible outcomes}}$$

Ex 1 How many 5 card hands are possible?

$$_{52}C_5 = 2{,}598{,}960$$

Ex 2 How many 5 card hands can be formed that are just hearts?

$$_{13}C_5 = 1287$$

$$P(\text{FLUSH}) = \frac{1287}{2598960} = .000485 \text{ ish}$$

or .0485%

Ex 3 All face cards?

$$_{12}C_5 = 792$$

Ex 4 5 card hands that have 3 hearts and 2 spades?

$$_{13}C_3 \cdot {}_{13}C_2 = 22308$$

Ex 5 Full House?

3 of a kind & 2 of a kind?

$$(_{13}C_1 \cdot {}_3C_2) \cdot ({}_{12}C_1 \cdot {}_3C_1) = \cancel{2197}$$

(3 of a kind) (2 of a kind)

$$13 \cdot 3 \cdot 12 \cdot 3 = 1404$$

$$\frac{1404}{2598960} = .0005$$

Evaluate $\dfrac{100!}{3!97!} = 161700$

$$\frac{100 \cdot \overset{33}{\cancel{99}} \cdot \overset{49}{\cancel{98}} \cdot \cancel{97!}}{\cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{97!}}$$

Try $_nC_2 = 21$ find n

$$\frac{n!}{(n-2)!2!} = 21$$

$$\frac{n(n-1)\cancel{(n-2)!} \cdot \cancel{2}}{\cancel{(n-2)!} \cdot \cancel{2}} = 21 \cdot 2$$

$$n^2 - n = 42 \implies n(n-1) = 42$$

$$n^2 - n - 42 = 0$$

$$(n-7)(n+6) = 0$$

$n = 7$ or $\cancel{n = 6}$