Combinatorics Test
Math 12

Name: Key

1. A couple is planning an evening out. They have a choice of 4 restaurants for dinner, 6 movies following dinner, and 4 coffee shops for after the movie. How many different ways can they plan the evening if they choose one of each?

$4 \cdot 6 \cdot 4 = 96$ /2

2. How many different ways are there to arrange the letters in the following words?

a. TSAWWASSEN $\frac{10!}{3!2!2!} = 151200$ /2

b. BLAKE STITT $\frac{10!}{3!} = 604800$ /2

3. The winner of a lottery chooses 5 vehicles from a warehouse that contains 13 different cars, 9 different trucks and 6 different motorcycles.

a. How many different choices of 5 vehicles are possible?

13+9+6

$28C_5 = 98280$ /2

b. How many different choices of 5 vehicles are possible if there must be at least one car?

$98280 - \text{ZERO CARS}$

$98280 - 15C_5 = 95277$ /2

10

4. Codes with 5 digits are made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. If repetitions are NOT allowed and each code must contain 3 odd digits followed by 2 even digits, determine the number of different codes that can be made.

O O O E E

$5 \cdot 4 \cdot 3 \cdot 4 \cdot 3 = 720$

60

/2

5. A class of 16 students is made up of 7 girls and 9 boys. From this class, a group of 5 students is chosen to represent the class at a competition.
   a. Determine the number of different groups of 5 that can be formed if there must be 2 girls and 3 boys in each group.

$7C_2 \cdot 9C_3 = 1764$

/2

   b. Determine the number of different groups of 5 that can be formed if there must be at most 1 boy in each group.

0B or 1B

$7C_5 + 7C_4 \cdot 9C_1 = 336$

/2

/6

6. A postal code consists of three letters and three digits arranged with a letter first, then a digit, a letter, then a digit, and a letter and a digit. If the first letter must be V, W, or X and there are no other restrictions on the other letters or digits, determine how many different postal codes are possible. An example of a postal code is V0N 5Y2.

$3 \cdot 10 \cdot 26 \cdot 10 \cdot 26 \cdot 10$

$= 2{,}028{,}000$ /2

7. A class of 34 students consists of 20 girls and 14 boys. How many different committees of 5 girls and 3 boys can be formed from this class?

$20C_5 \cdot 14C_3 = 5643456$ /2

8. In a library, 5 different mystery novels, 4 different reference books and 3 different science fiction novels are to be arranged on a shelf.
   a. How many ways can you arrange the books on the shelf?

$(5!4!3!)6!$ $12! = 479{,}001{,}600$ /2

   b. How many different arrangements are there for the 12 books if the books on each subject must be kept together?

$5!4!3!\cdot 3! = 103680$ /2

/8

9. In a standard deck of 52 cards, how many different 4-card hands are there that contain at most 1 spade?

13 Spades

$${}_{39}C_4 + {}_{39}C_3 \cdot {}_{13}C_1 = 201058$$ /2

10. In a standard deck of 52 cards, how many different 5-card hands are there that contain at most 3 face cards?

0 + 1 + 2 + 3 or Total - 4 - 5

$${}_{40}C_5 + {}_{12}C_1 \cdot {}_{40}C_4 + {}_{12}C_2 \cdot {}_{40}C_3 + {}_{12}C_3 \cdot {}_{40}C_2$$

$$658008 + 1096680 + 652080 + 171600$$

$$2578368$$ /2

11. Solve algebraically using factorial notation: ${}_nP_2 = 90$

$$\frac{n!}{(n-2)!} = 90$$

$$n(n-1) = 90$$

$$n^2 - n - 90 = 0$$

$$(n-10)(n+9) = 0$$

$n = \cancel{-9}, 10$

rej

$n = 10$ /3

12. Determine the 7th term in the expansion of $(2x - y)^{11}$

$n = 11$

$k = 6$

$a = 2x$

$b = -y$

$${}_{11}C_6 (2x)^5 (-y)^6$$

$$= 462(32x^5)y^6$$

$$= 14784x^5y^6$$ /2

/9

13. Determine the coefficient of the 4th term of the expansion of $(x+2y)^7$

$n=7$
$k=3$
$a=x$
$b=2y$.

${}_7C_3\, x^4(2y)^3$

$35x^4 \cdot 8y^3$

$280x^4y^3$

280

/2

14. In the expansion of $(a^2-b)^4$, determine the middle term.

term 3
$n=4$
$k=2$
$a=a^2$
$b=-b$

${}_4C_2 (a^2)^2(-b)^2$

$= 6a^4b^2$

/2

15. Evaluate: $\frac{250!}{247!}$

$250 \cdot 249 \cdot 248 = 15{,}438{,}000$

/2

16. Simplify the following expression without using the factorial symbol (in the final answer).

$$\frac{(n-2)!(n+1)!}{(n!)^2}$$

$$\frac{(n-2)!(n+1)(n)!}{(n)(n-1)(n-2)!\,n!} = \frac{n+1}{n^2-n}$$

or $\frac{n+1}{n(n-1)}$

/2

/8

17. Solve: $\frac{n!}{(n-2)!3!} = 5$

$n = 6$

$n(n-1) = 30$

$n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$

$n = -5, 6$

rej $(n > 0)$

3

18. Given Pascal's triangle below, which of the following is equivalent to the value of x ?

$_0C_0$ 1

$_1C_0$ 1 1

$_2C_0$ — — —

$_3C_0$ — — — —

$_4C_0$ — — — — —

$_5C_0$ — — x — — —

A. $_4C_2$

B. $_5C_2$

C. $_6C_2$

D. $_6C_3$

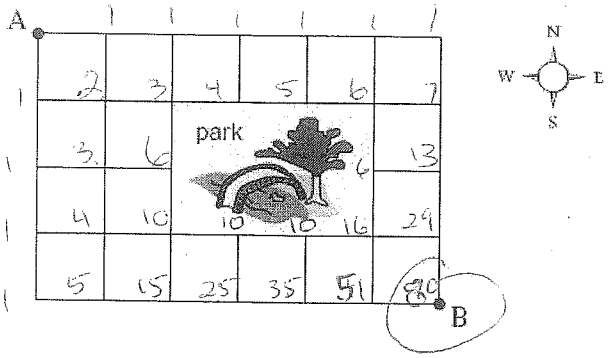
$_6C_2$

1

4

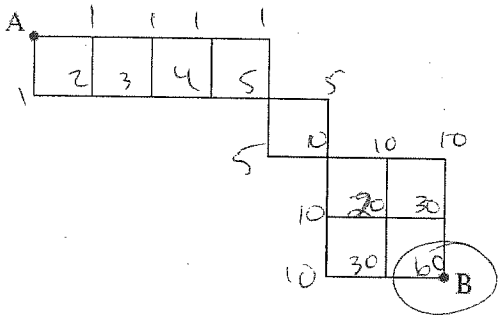
#19 & 20 You MUST show work → just the answer alone will NOT get you marks!

19. The diagram below represents a street map. If a person can only travel east or south on the streets, how many different routes are there from A to B?



80

20. Moving only to the right or down, determine the number of different pathways from A to B.



60