$\boxed{\text{DERIVATIVES!!}}$

Derivatives from "FIRST PRINCIPLES"

A DERIVATIVE of a f^n, $f(x)$, at a pt (a, y) is:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

(f prime of a)

(if the derivative exists!)

OR

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Ex 1 $f(x) = 2x^2 - 5x + 6$ (4)

want to find $f'(4)$ so a is 4

$$f'(4) = \lim_{x\to 4} \frac{(2x^2-5x+6)-(18)}{x-4}$$

$$= \lim_{x\to 4} \frac{2x^2-5x-12}{x-4}$$

$$= \lim_{x\to 4} \frac{(2x+3)\cancel{(x-4)}}{\cancel{(x-4)}} = \underline{\underline{11}}$$

In general, the derivative of a f^n at any pt $a = x$,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Interpretations of the Derivative

- slope of the tangent
- as a rate of change
  - $\Rightarrow$ The rate of change of $y = f(x)$ w.r.t x when $x = a$ is $f'(a)$

Ex Rate of change of volume as the radius increases.

Ex 2 Find the derivative of $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$x^2 - 4$

$$= \lim_{h\to 0} \frac{(x+h-x)(x+h+x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x$$

$f'(3) = 2(3) = 6$

$f'(-5) = -10$

Ex 3 If $f(x) = \sqrt{x+2}$, find $f'(x)$

& State the Domains

$$f'(x) = \lim_{h\to 0} \frac{(\sqrt{x+h+2}) - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{(\sqrt{x+h+2}+\sqrt{x+2})}$$

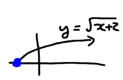
$f(x+h)$ $-$ $f(x)$

$$= \lim_{h\to 0} \frac{x+\cancel{h}+2 - x - 2}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})} = \frac{1}{2\sqrt{x+2}}$$

$\sqrt{x+2} + \sqrt{x+2}$

$f(x)$ Domain: $x \geq -2$

$f'(x)$ Domain $x > -2$



Newtonian

$f'(x)$ is the derivative

$f'(3) \Rightarrow$ is the value of the derivative at $x=3$

Calcish (Leibniz-ian)

the derivative of $f(x)$ is:

$\frac{dy}{dx}$ is NOT a fraction, its a symbol

comes from

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

$\Rightarrow$ the derivative of y w.r.t. x

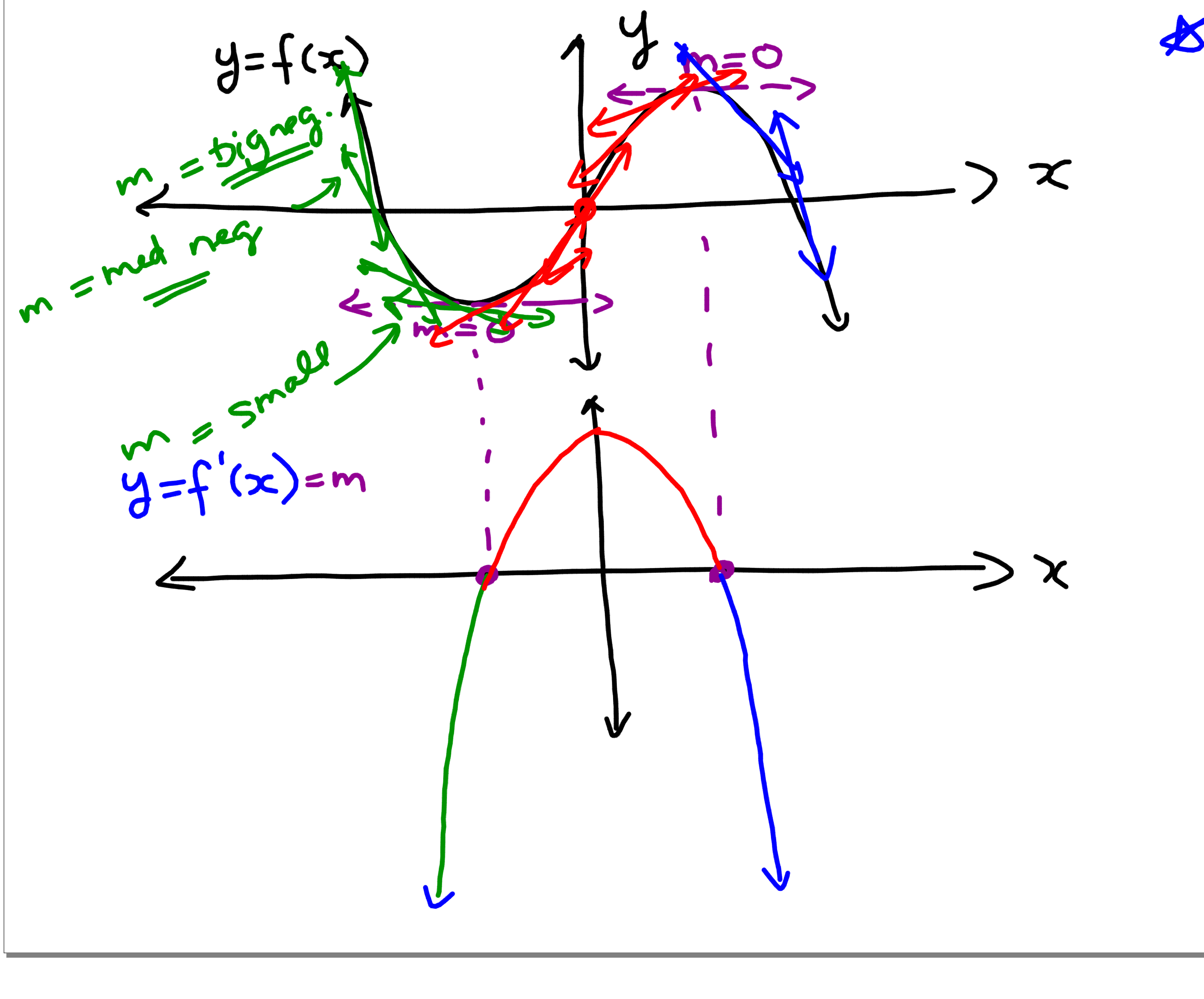
$\frac{d}{dx}(f(x))$

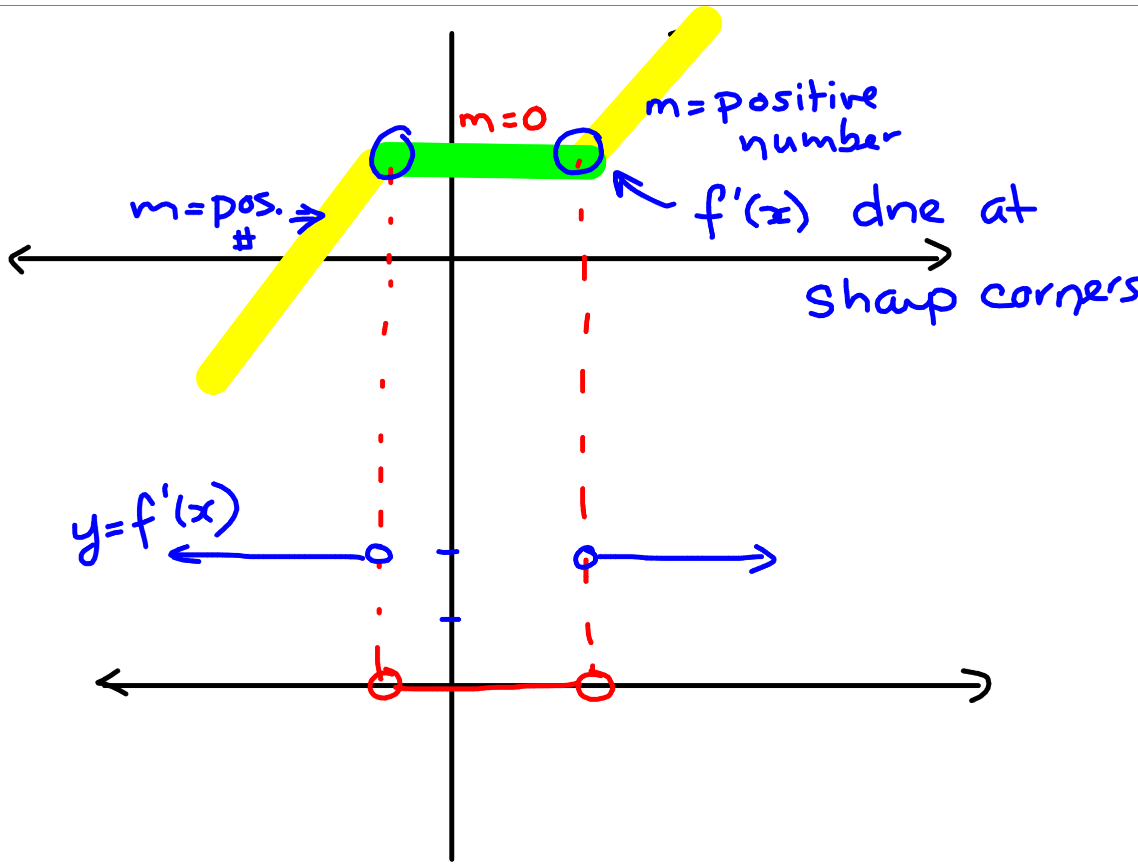
$\frac{d}{d}$

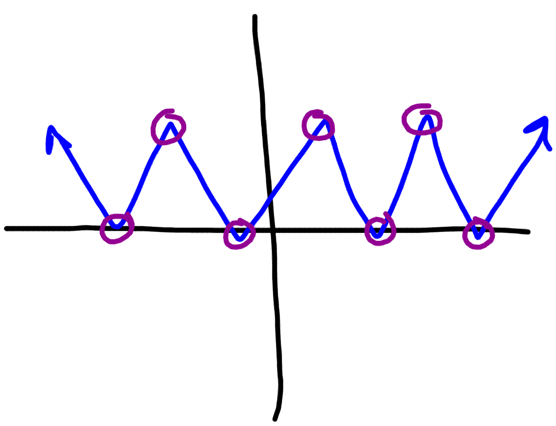
$f'(3) \longleftrightarrow \left.\frac{dy}{dx}\right]_{x=3}$

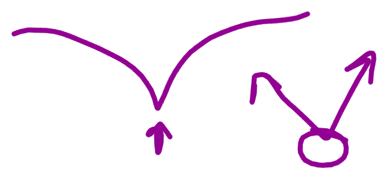
Using $f(x)$ to sketch $f'(x)$

Remember: $f'(x)$ is the SLOPE of the tangent line to $f(x)$









7 pts where $f'(x)$ dne

$f'(x)$ dne where $\lim_{h \to 0}$ dne

however: the limit can exist where $f'(x)$ does not.