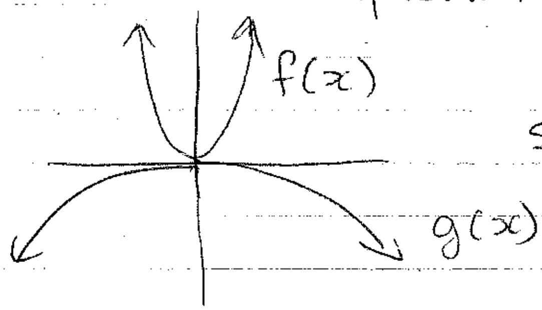
PreCalculus 12 Practice Final

__A__ #1 $g(x) = (x-4)^2$
↑ 4 units Right so (A)

__A__ #2 pt A (5, −1) move it 3 up
A $(5,-1) \rightarrow (5, -1+3) \rightarrow (5, 2)$ (A)

__A__ #3 $f(x) = x^2$ is a parabola
$f(x) = -\frac{1}{5}x^2$ → flipped over x-axis
→ all y-values mult by $\frac{1}{5}$ so it is flatter

∴ 

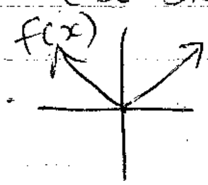
so (A)

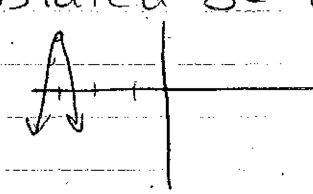
__C__ #4 $g(x) = f(2x-4)$ (RED FLAG → Factor first!)
$= f(2(x-2))$ so: • x values ÷2 or a horizontal stretch by a factor of $\frac{1}{2}$

• translated 2 units Right

__C__ #5 $g(x) = (x-6)^2 - 3$
↑ 6 Right ↑ 3 down

__C__ #6 $g(x) = -4|x+3| + 2$
• Reflected across x axis, y values mult. by 4 (so "skinnier"), translated 3 ← and 2 ↑

∴ 

becomes 

A #7 $f(x) = 3x+5$

$y = 3x+5$

$x = 3y+5$ (x + y switch places)

$-5 \quad -5$ (solve for y)

$\frac{x-5}{3} = \frac{3y}{3}$

$y = \frac{x-5}{3}$

$f^{-1}(x) = \frac{x-5}{3}$ or $f^{-1}(x) = \frac{1}{3}x - \frac{5}{3}$

D #8* $(9,-10)$ but $(9,-10)$ is not on

$\times 3$

$(9,-30)$ $y = \sqrt{x}$ so this is a very silly question!!

B #9 ODD-DEGREE go up on one side & down on the other (or) so it has to be B or D, but D has 3 x-intercepts so it is B.

D #10 Degree of 4 (from ax^4) means up to 4 solutions / x intercepts

D #11 Restriction comes from the divisor or $x-2$ So $x \neq 2$

B #12

Root of $(x-3)$

$$
\begin{array}{r|rrrr}
3 & -8 & -8 & -8 & -8 \\
  &   & -24 & -96 & -312 \\
\hline
  & -8 & -32 & -104 & -320
\end{array}
$$

↑ Remainder

D #13 If $P(6)=0$ then $+6$ is a root (solution) of $P(x)$ so $\underline{x-6}$ is a factor

B #14 Test factors of 35 so $\pm 1, \pm 5, \pm 7, \pm 35$

D #15 If $x-a$ is a factor of $f(x)$, then $f(a)=0$

so: $x-2 \rightarrow f(2) = 2^3 + 12(2)^2 + 29(2) + 18 \neq 0$ so $x-2$ NOT a factor

$x-1 \rightarrow f(1) = 1^3 + 12(1)^2 + 29(1) + 18 \neq 0$

$x-9 \rightarrow f(9) \neq 0$

$x+2 \rightarrow f(-2) = (-2)^3 + 12(-2)^2 + 29(-2) + 18$

$= -8 + 48 - 58 + 18$

$= 0$ ✓

so $x+2$ is a factor

D #16 If $x+7$ is a factor, then $f(-7)=0$

so $(-7)^3 + 21(-7)^2 + 111(-7) + K = 0$

$-343 + 1323 - 777 + K = 0$

$-91\,\cancel{203} + K = 0$ oops

so $\underline{K = \cancel{+203}\ 91}$

$K = 91$

A #17 multiplicity of 2 means it touches the x-axis without crossing it, like the vertex of a parabola.

A #18 $\frac{255}{360} = \frac{x}{2\pi}$ or $\frac{255}{180} \cdot \pi = \frac{x}{\cancel{\pi}} \cdot \cancel{\pi}$

$$x = \frac{255}{180}\pi$$

$$x = \frac{17}{12}\pi$$

D #19 $\frac{11}{6}\pi$ is in Q$\underline{IV}$ so $\underline{D}$

B #20 $-5000 \div 360 = -13.88$

~~[illegible]~~ so -13 revolutions

$-13 \times 360 = -4680$

$-5000 - (-4680) = -320°$ left over

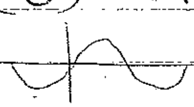
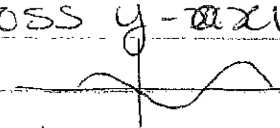
$$\frac{-320}{360} = -\frac{8}{9}$$

so: $-13\frac{8}{9}$ revolutions

B #21 amplitude is $\underline{2}$ (always positive)

period is $\frac{360}{5} = \underline{\underline{72°}}$

C #22 $(-\theta)$ → reflects across y-axis

 becomes 

(no phase shift)

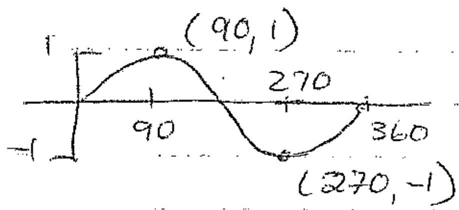
D #23 amplitude is 7

A #24 Phaseshift is $\frac{\pi}{3} \rightarrow$ (right)

C #25 $\sin^2 x - 1 = 0$

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$



$x = 90°$ and $270°$

D #26 radius is 17

B #27 period $= \dfrac{2\pi}{\frac{\pi}{110}} = 2\cancel{\pi} \cdot \dfrac{110}{\cancel{\pi}} = \underline{220 \text{ seconds}}$

C #28 minimum height is the sinusoidal axis minus the amplitude

or $20 - 17 = \underline{3}$

D #29 max height: $20 + 17 = \underline{37\text{ m}}$

B #30 $\cos x = -\frac{3}{5}$

$-5\cos x - 3 = 0$

$+3 \quad +3$

so QII & QIII

$\dfrac{-\cancel{5}\cos x}{-\cancel{5}} = \dfrac{3}{-5}$

$\measuredangle x = \cos^{-1}(3/5) = .9273$ (in QI or as the ref $\measuredangle$)

QII: $x = \pi - .9273 = \underline{2.214}$

QIII: $x = \pi + .9273 = \underline{4.069}$

A #31 $\cot\theta \sin\theta \sec\theta =$

$$\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}} = 1$$

C #32 $$\frac{\cos\theta}{1+\sin\theta} \cdot \frac{(1-\sin\theta)}{(1-\sin\theta)}$$ (mult by 1 in the form of conjugate pairs)

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$ (~~fox~~ multiplied $(1+\sin\theta)(1-\sin\theta)$)

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$ (Pyth. identity)

$$= \frac{1-\sin\theta}{\cos\theta}$$ (reduced $\frac{\cos\theta}{\cos^2\theta}$ to $\frac{1}{\cos\theta}$)

B #33 $\tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$ (replace $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$)

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \cdot \frac{\cos^2\theta}{\cos^2\theta}$$ (make a common denom.)

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$ (factor out $\sin^2\theta$)

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$ ($\frac{\sin^2\theta}{\cos^2\theta} = \tan\theta$ and $1-\cos^2\theta = \sin^2\theta$)

$$= \tan^2\theta \sin^2\theta$$

B #34 use a sum/difference identity

$$\cos x\cos y - \sin x\sin y = \cos(x+y)$$

C #35 $2\cos x\cos 2x - 2\sin x\sin 2x = -1$

$$\Rightarrow 2(\cos x\cos 2x - \sin x\sin 2x) = -1$$

$$2(\cos(x+2x)) = -1$$

$$2\cos 3x = -1$$

use technology $\Rightarrow$ graph $y_1 = 2\cos 3x$

$y_2 = -1$ ~~and~~

and get the solutions or!

$\cos(3x) = -\frac{1}{2}$ (in Q II and Q III)

$(3x) = 60° \rightarrow$ in Q III: $3x = 240$

$x = 80°$

in Q II:

$3x = 120$

$x = 40°$

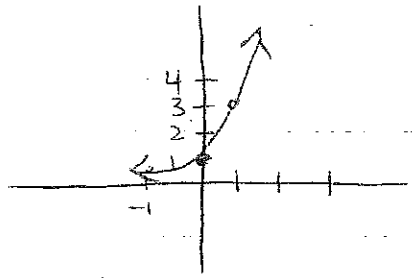
Next solutions are one period later

period $= \frac{360°}{3} = \underline{\underline{120°}}$

So General sol$\underline{^n}$:

$$x = \begin{cases} 40° + 120°n \\ 60° + 120°n \end{cases} \quad \text{where } n \in I$$

B #36 $y = 3^x$



A #37* $A = 1\left(\frac{1}{2}\right)^{t/11}$ (The original wording was weird → change by removing "in 11-day intervals)

D #38 $P = 750(3)^t$

D #39 $\log 7333 = \log 5^x$ (log both sides)

$$\frac{\log 7333}{\log 5} = x\frac{\cancel{\log 5}}{\cancel{\log 5}}$$

$x = 5.5299$

C #40 $(64)^{3x} = 512^{(x+7)}$

$(8^2)^{3x} = (8^3)^{x+7}$

$8^{6x} = 8^{3x+21}$

$\therefore \quad 6x = 3x + 21$

$\quad -3x \quad -3x$

$3x = 21$

$\underline{x = 7}$

D #41 $\log_4 1024 = 5$

C #42 $2^b = 4$

A #43 $4^{-2} = \frac{1}{16}$

$\log_4\left(\frac{1}{16}\right) = -2$

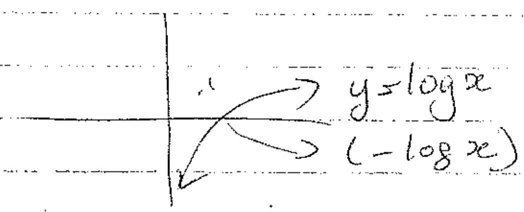
D #44 $\log_4 65536$

$= \frac{\log 65536}{\log 4}$ (change of base rule)

$= 8$

A #45 oops RANGE!

$y \in \mathbb{R}$



D #46 $\frac{6(x-9)}{6} > \frac{0}{6}$

$x - 9 > 0$

$x > 9$

A #47 $\log \frac{5}{441} = \log \frac{5}{9 \cdot 49} = \log 5 - (\log 9 + \log 49)$

$= \log 5 - (\log 3^2 + \log 7^2)$

$= \log 5 - (2\log 3 + 2\log 7)$

$= \log 5 - 2(\log 3 + \log 7)$

$= u - 2(r + z)$

A #48 $\log_4 5w^{10}y = \log_4 5 + \log_4 w^{10} + \log_4 y$

$= \log_4 5 + 10\log_4 w + \log_4 y$

A #49 $\log_8 25^6 - (\log_8 24)$

$= \log_8 \frac{25^6}{24} = \frac{\log(25^6/24)}{\log 8}$

$= 7.7594$

C #50 $\log_3 x = \log_3 3 + \log_3 2$

$\log_3 x = \log_3 6$

$\therefore x = 6$

A #51 $2^{8x-3} = (2^2)^{x+2}$

$2^{8x-3} = 2^{2x+4}$

$\therefore 8x-3 = 2x+4$

$-2x \quad -2x$

$6x - 3 = 4$

$+3 \quad +3$

$6x = 7$

$x = \frac{7}{6} = 1.17$

B #52 $(x^2-8) + (-2-x^2)$

$= -10$

B #53 $\frac{x^3-81x}{x+9} = x\frac{(x^2-81)}{x+9} = \frac{x(x-9)\cancel{(x+9)}}{\cancel{(x+9)}}$

$(x \neq -9)$

$= x(x-9) \quad x \neq -9$

C #54 $(f \circ g)(x) = f(g(x))$

$(f \circ g)(x) = \left(\frac{1}{x-3}\right) + 3$
$(x \neq 3)$

$= \frac{1}{x-3} + \frac{3(x-3)}{x-3}$

$= \frac{1 + 3x - 9}{x-3}$

$= \frac{3x-8}{x-3} \quad x \neq 3$

D #55 $5(2-x) - (9x^2+7x)$
$= 10 - 5x - 9x^2 - 7x$

$-9x^2 - 12x + 10.$

B #56 $f(f(8)) = f(55) = 55^2 - 9 = 3016.$

$f(8) = (8)^2 - 9$
$= 55$

D #57 $9! = 362880$.

A #58 $[(5!)(3!)(2!)]3!$

$= 8640$

D #59 ${}_{11}C_8 = 165$

A #60 ${}_9C_8 \cdot {}_{12}C_6 = 9 \cdot 924$
$= 8316$

C #61 If x has a power of 5, it is the 8^{th} term:

using $t_{k+1} = {}_nC_k\, a^{n-k} b^k$

n is 12 so if the exponent is 5, then $12-7=5$, so $k=7$

i.e. $n=12$
$k=7$
$a=x$
$b=y$

${}_{12}C_7\, x^7 b^5$

$\downarrow$

${}_{12}C_7 = 792$

D #62 $a = x$
$b = -2$
$n = 9$
$k = 5$

${}_9C_5\, x^4 (-2)^5$

$\downarrow$

$126 \cdot x^4 \cdot -32$

$= -4032x^4$

B #63 $(-4x-2y)^{29}$ has 30 terms.

A #64 $(7!)(32!)$

WRITTEN - Short Answer

1. $\log(3x+15) = 1 + \log(x+3)$

$\log(3x+15) - \log(x+3) = 1$

$\log\frac{(3x+15)}{(x+3)} = 1$ (now write in exponential form)

$(x+3)10^1 = \frac{(3x+15)}{\cancel{(x+3)}} \cancel{(x+3)}$

$10x + 30 = 3x + 15$
$-3x \quad\quad -3x$
$7x + 30 = 15$
$-30 \quad -30$
$\frac{7x}{7} = \frac{-15}{7}$

$x = \frac{-15}{7}$ (check in original equation ✓)

2.(a) $f(x) = \frac{5}{2}x - 3$

$y = \frac{5}{2}x - 3$ (switch x & y)

$x = \frac{5}{2}y - 3$ (solve for y)

$+3 \qquad +3$

$2(x+3) = 2 \cdot \frac{5}{2}y$

$\frac{2x+6}{5} = \frac{5y}{5}$

$y = \frac{2x+6}{5}$ or $\frac{2x}{5} + \frac{6}{5}$

(b) $y = 3(x-2)^2 - 3$

$x = 3(y-2)^2 - 3$

$\frac{x+3}{3} = \frac{3(y-2)^2}{3}$

$\sqrt{\frac{x+3}{3}} = \sqrt{(y-2)^2}$

$y - 2 = \pm\sqrt{\frac{x+3}{3}}$

$y = \pm\sqrt{\frac{x+3}{3}} + 2$

3. (a) $3x^4 - 48 = 0$

$3(x^4 - 16) = 0$

$3(x^2+4)(x^2-4) = 0$

$3(x^2+4)(x-2)(x+2) = 0$

$x = 2, -2$

(b) $-x^3 + 12x^2 - 47x + 60 = 0$

Find 1st root by using the remainder theorem & factors of <u>60</u>

$P(1) = -1 + 12 - 47 + 60 \neq 0$

$P(-1) = 1 + 12 + 47 + 60 \neq 0$

$P(2) = -8 + 48 - 94 + 60 \neq 0$

$P(-2) = 8 + 48 + 94 + 60 \neq 0$

$P(3) = -27 + 108 - 141 + 60 = 0$ ✓

so $(x-3)$ is one factor.

$$\begin{array}{r|rrrr} 3 & -1 & 12 & -47 & 60 \\ & \downarrow & -3 & 27 & -60 \\ \hline & -1 & 9 & -20 & 0 \end{array}$$

$(x-3)(-x^2+9x-20) = 0$

$(x-3)(-x+4)(x-5) = 0$

so $x = \underline{\underline{3, 4 \text{ or } 5}}$

c) $21x^4 - 7x^3 - 6x^2 + 2x = 0$

$x(21x^3 - 7x^2 - 6x + 2) = 0$

$P(1) = 21(1) - 7(1) - 6(1) + 2 \neq 0$
$P(-1) = -21 - 7 + 6 + 2 \neq 0$
$P(2) = 168 - 28 - 12 + 2 \neq 0$
$P(-2) = -168 - 28 + 12 + 2 \neq 0$

...hmm.

$(21x^3 - 7x^2) - (6x \mp 2)$ try factoring by grouping

$7x^2(3x-1) - 2(3x-1)$

$= (3x-1)(7x^2-2)$

so $21x^4 - 7x^3 - 6x^2 + 2x = 0$

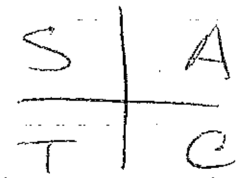
$x(3x-1)(7x^2-2) = 0$

$\downarrow \quad \downarrow \quad \downarrow$

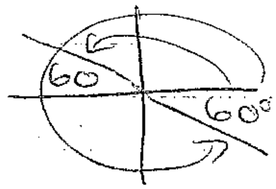
$\underline{x=0}$ $\quad$ $\underline{x = \frac{1}{3}}$ $\quad$ $x^2 = \frac{2}{7}$

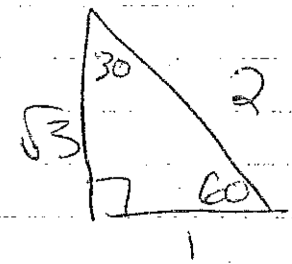
$\underline{x = \pm\sqrt{\frac{2}{7}}}$

4. $\tan\theta = -\sqrt{3}$ in QII + IV



Ref $\angle = 60°$





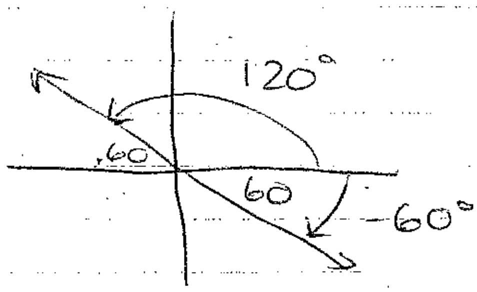
so $\theta = 120°$ and $300°$

only domain is

$-180 \leq \theta \leq 180°$

$\tan 60 = \frac{\sqrt{3}}{1}$

so:



$\therefore \theta = -60°$ or $120°$

5. $6\sin^2 x - 5\cos x - 2 = 0$ $\quad [0, 2\pi]$

$6(1-\cos^2 x) - 5\cos x - 2 = 0$

$6 - 6\cos^2 x - 5\cos x - 2 = 0$

$-6\cos^2 x - 5\cos x + 4 = 0$

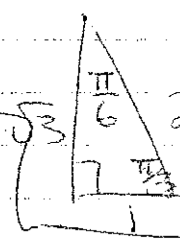
$6\cos^2 x + 5\cos x - 4 = 0$

$(3\cos x + 4)(2\cos x - 1) = 0$

$\downarrow$ $\qquad\qquad$ $\downarrow$ $\qquad$ (Q I & IV)

$\cancel{\cos x = -\frac{4}{3}}$ $\qquad\qquad$ $\cos x = \frac{1}{2}$

reject because $\cos x$ reaches a minimum of -1 & never gets to $-\frac{4}{3}$



$$\boxed{\begin{array}{l} x = \frac{\pi}{3} \ (\text{Q I}) \\ \text{or } x = \frac{5\pi}{3} \ (\text{Q IV}) \end{array}}$$

6. $h(x) = \dfrac{x^2-4}{x^2-3x+2} = \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} \quad x \neq 1, 2$

$$h(x) = \frac{x+2}{x-1} \quad x \neq 1, 2$$

7. At least 1 man and 1 woman (5M 8W, 5 person committ

Means: 1 Man 4 W
2 Men 3W
3 m 2 W
4 m 1 W

$$_5C_1 \cdot {_8C_4} + {_5C_2} \cdot {_8C_3} + {_5C_3} \cdot {_8C_2} + {_5C_4} \cdot {_8C_1}$$

$$350 + 560 + 280 + 40$$

$$= \underline{\underline{1230}}$$

Alternate Solution

# of ways to make 5 person committee − 0 Men − 0 women

$$_{13}C_5 - {_8C_5} - {_5C_5}$$

$$1287 - 56 - 1 = \underline{\underline{1230}}$$

PROBLEMS

$1 + \cos\theta$	$= \dfrac{\sin^2\theta}{1-\cos\theta}$
	$= \dfrac{1-\cos^2\theta}{1-\cos\theta}$
	$= \dfrac{(1-\cancel{\cos\theta})(1+\cos\theta)}{(1-\cancel{\cos\theta})}$
$1+\cos\theta$	$= 1+\cos\theta$

LS = RS QED

2. $$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \sec 2\theta - \tan 2\theta$$

$$= \frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{1-2\sin\theta\cos\theta}{\cos 2\theta}$$

$$= \frac{1-2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$$

$$= \frac{1-2\sin\theta\cos\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{\overbrace{\sin^2\theta+\cos^2\theta}^{1}-2\sin\theta\cos\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)\overset{-1}{\cancel{(\sin\theta-\cos\theta)}}}{\cancel{(\cos\theta-\sin\theta)}(\cos\theta+\sin\theta)}$$

$$= \frac{-(\sin\theta-\cos\theta)}{(\cos\theta+\sin\theta)}$$

$$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

LS = RS

3. $\dfrac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta} = \tan\theta$

$1=\sin^2\theta+\cos^2\theta$

$$\frac{1-(\cos^2\theta-\sin^2\theta)+2\sin\theta\cos\theta}{1+(\cos^2\theta-\sin^2\theta)+2\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cancel{\cos^2\theta}-\cancel{\cos^2\theta}+\sin^2\theta+2\sin\theta\cos\theta}{\cancel{\sin^2\theta}+\cos^2\theta+\cos^2\theta-\cancel{\sin^2\theta}+2\sin\theta\cos\theta}$$

$$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$$

$$=\frac{\cancel{2}\sin\theta\,(\cancel{\sin\theta+\cos\theta})}{\cancel{2}\cos\theta\,(\cancel{\cos\theta+\sin\theta})}$$

$$=\frac{\sin\theta}{\cos\theta}$$

$=\tan\theta$ $\qquad$ $=\tan\theta$

LS = RS

$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$$

(factor) $= \dfrac{(\cos\theta - \sin\theta)\cancel{(\cos\theta + \sin\theta)}}{\cos\theta\cancel{(\cos\theta + \sin\theta)}}$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= 1 - \tan\theta = 1 - \tan\theta$$

LS = RS ü