

## Combinatorics

aka Advanced Counting ü

### Fundamental Counting Principle

E    H    Egg or Ham

M   J   C    Milk or Juice or Coffee

Y   IC    Yogurt or Ice Cream

(a) One item from each category: List all possible meals.

(12) ? EMY   EMIK   EJY   EJK   EKY   EKIC  
HMY   HMIK   HJY   HJK   HCY   HCIC

12 possible meals

$$\text{OR } \begin{array}{c} \underline{2} \\ \uparrow \\ EH \end{array} \times \begin{array}{c} \underline{3} \\ \uparrow \\ MJC \end{array} \times \begin{array}{c} \underline{2} \\ \uparrow \\ YIC \end{array} = 12$$

(b) Another cafe has 24 flavours of ice cream. You can order regular waffle or sugar cones. You can order a double cone with 2 scoops

(1) Types of cone: 3

(2) # of choices for scoop 1: 24

(3) # of choices for scoop 2: 24

# of possible cones:

$$3 \times 24 \times 24 = \underline{\underline{1728}} \text{ possibilities}$$

TRY:  
 • 7 different hard drives  
 • 3 different monitors  
 • 5 different printers  
 • 2 multimedia packages

$$\text{How many? } 7 \times 3 \times 5 \times 2 = \underline{\underline{210}}$$

The Fundamental counting principle:

If one item can be selected in  $m$  ways and a second item can be selected in  $n$  ways, then the two items can be selected in  $m \cdot n$  ways.

Examples

(1) How many different 2 digit numbers are there?

$$\begin{array}{c} 9 \times 10 = \underline{90} \\ \uparrow \quad \uparrow \\ \text{9 choices} \quad \text{10 choices} \\ 1, 2, \dots, 9 \quad 0, 1, 2, \dots, 9 \end{array}$$

(2) How many ODD 3 digit #'s are there?

$$\begin{array}{c} 223 \\ \uparrow \\ 9 \times 10 \times 5 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{9 choices} \quad \text{5 choices} \quad 1, 3, 5, 7, 9 \end{array}$$

450 ODD 3 digit #'s

(3) How many 4 digit #'s can be formed if REPETITION is NOT allowed

$$\begin{array}{c} 9 \times 9 \times 8 \times 7 = \underline{4536} \\ \uparrow \quad \uparrow \\ \begin{array}{c} 1, 2, 3, 4, 5 \\ 6, 7, 8, 9 \end{array} \quad \begin{array}{c} 1, 2, 3, 4 \\ 5, 6, 7, 8, 9 \end{array} \end{array}$$

$$\begin{aligned} (4) \quad & \frac{26 \cdot 26 \cdot 26}{L \ L \ L} \cdot \frac{10 \cdot 10 \cdot 10}{d \ d \ d} \\ & = 17,576,000 \end{aligned}$$

(5) A Multiple Choice test has 30 questions, 4 possible answers for each. How many possible answer keys are there?

$$\begin{array}{c} 4 \times 4 \times 4 \times 4 \times 4 \times 4 \dots 4 \\ \text{or } 4^{30} = 1.153 \times 10^{18} \end{array}$$

$$\begin{array}{c} 1 \\ \hline 1153000000000000000 \end{array}$$