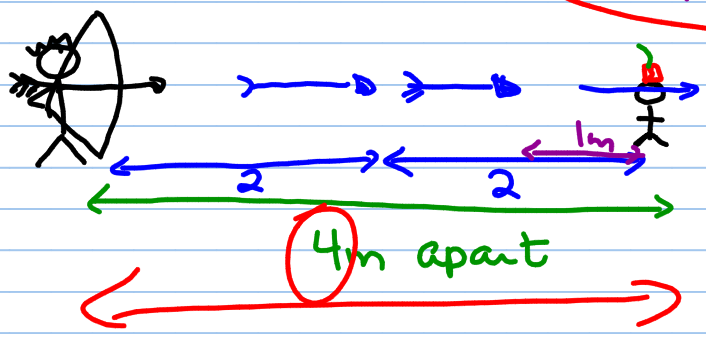
Infinite Series

Xeno's Paradox:



$+\frac{1}{2}, +\frac{1}{4}, +\frac{1}{8} + \frac{1}{16}, \ldots + \frac{1}{128}, \ldots$

$\frac{1}{32768}$

$1 + 2 + 4 + 8 + 16 + 32 + \ldots$

A Geometric Series is said to be CONVERGENT if:

→ the terms are getting smaller and smaller and smaller and smaller ...

→ This happens if the common ratio is between -1 and 1

$$-1 < r < 1$$

$$|r| < 1$$

→ If the Series is Convergent then it has a finite sum

$$S_\infty = \frac{t_1}{1-r}$$ only works if $-1 < r < 1$

Ex: $t_1 = 2$, $r = \frac{1}{2}$

$$S_\infty = \frac{2}{1-\frac{1}{2}} = \frac{2}{\frac{1}{2}} = 2 \cdot \frac{2}{1}$$

$$= 4$$

If r is bigger than 1 or less than -1, $r > 1$ or $r < -1$, then the Series is DIVERGENT and has no finite sum.

Ex 2 $10 - \frac{10}{3} + \frac{10}{9} - \ldots$

$S_\infty = \frac{t_1}{1-r}$

$t_1 = 10$

$r = -\frac{1}{3}$

So $-1 < r < 1$ is true

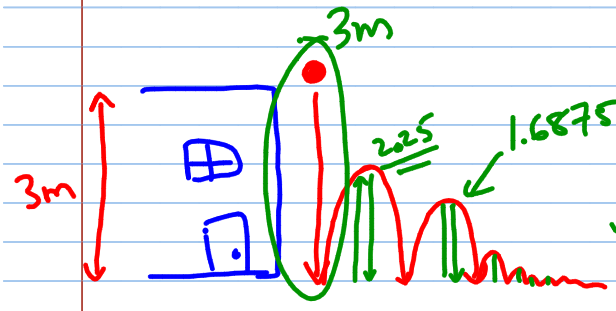
$\therefore$ "Therefore" the series is convergent and we can use our formula

$$S_\infty = \frac{10}{1-(-\frac{1}{3})} = \frac{10}{\frac{3}{3}+\frac{1}{3}} = \frac{10}{\frac{4}{3}}$$

$$= \frac{10}{1} \times \frac{3}{4} = \frac{30}{4}$$

$$= \frac{15}{2} \text{ or } 7.5$$

A Ball is dropped from a height of 3 m. On each bounce it comes back to 75% of it's original height. If we let it keep bouncing until it stops (forever), what VERTICAL distance does it travel?



$t_1 = 3$
$r = 0.75$

$3 + 2.25 + 1.6875 + \ldots$ (2, 2)

Method 1:

$t_1 = 3$ $r = 0.75$ $S_\infty = \frac{3}{1-.75} = 12$ down

$\times 2$ up

24

-3 don't double the first one

21m

Method 2:

Use $t_1 = 2.25$ $r = .75$

$S_\infty = \frac{2.25}{1-.75} = 9$ up

$\times 2$ down

18

$+3$ add in the first drop

21 m