Introduction to Logarithms

$3^{7x} = 81^{2x-3}$ ... "cooked" (easy to solve)

$17 = 3^x$ ... we invented a new operation called a logarithm or "log"

$\Downarrow$

$\log_3 17 = x$

The answer to a log is the exponent you need on the base to get the argument

For example:

$\log_3 81 = 4$ since $3^4 = 81$

Logarithmic Form	Exponential Form
$\log_a b = x$	$a^x = b$
$\log_7 49 = 2$	$7^2 = 49$

Some log Rules

Since our number system is base 10,

$\log_{10} 100 = 2$

is written $\log 100 = 2$

assume base is 10 if it isn't stated otherwise

$e = 2.7182...$ $\log_e \Rightarrow \underline{\underline{\ln}}$

"logarithmus naturalis" or natural log

$\log_a 1 = 0$

$\log_a a = 1$

Change of base rule:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Example:

$$\log_3 17 = \frac{\log_{10} 17}{\log_{10} 3} = \log(17) / \log(3)$$

want $\log_{10}$

$= 2.578901923$

LOG RULES (Remember: the log is the exponent!)

Exp. Rules | Log Rules

$x^7 \cdot x^3 = x^{7+3} = x^{10}$

$\log(7\cdot 3) = \log 7 + \log 3$

$\log 21 \stackrel{?}{=} 0.845 + .477$

$1.3222 = 1.322$ ✓

$\log[ab] = \log a + \log b$

Ex: $\log_5 100 + \log_5 \frac{1}{4}$

$\log_5\left(100 \cdot \frac{1}{4}\right)$

$\log_5(25) = \underline{2}$

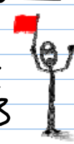
$\frac{x^7}{x^3} = x^{7-3} = x^4$

$\log\left[\frac{7}{3}\right]$

$= \log 7 - \log 3$

NOTE:

$\log\left[\frac{7}{3}\right] \neq \frac{\log 7}{\log 3}$



$(x^3)^7 = x^{21}$

$\log 3^7 = 7 \log 3$

$\log a^x = x \log a$

$\log_3 81$

$\log_3 3^4 = 4\boxed{\log_3 3} = 4$

$\log(a^x) = x \log a$

$(\log a)^x \Rightarrow \log^x(a)$

ONE LAST HELPFUL HINT

① $\log_a a^x = x$

Very handy

② $a^{\log_a x} = ? = x$

$\log_a \boxed{?} = \log_a \boxed{x}$

Ex $\log_5 5^{12} = 12$

$17^{\log_{17} 3} = 3$

① $\log_{211} 211^{13x+y}$ $\log_a a^x = x$

② $\log_3 24 - \log_3 8$

$= \log_3 \left[\frac{24}{8}\right] = \log_3 3 = 1$

③ $\log_2 3 + \log_2 12 - \log_2 9$

$= \log_2 \frac{3 \cdot 12}{9} = \log_2 4 = 2$

$$\log_a 3 + \log_a 5 + 1 - \log_a 6$$

$$\log_a 3 + \log_a 5 + \log_a a - \log_a 6$$

$$\log_a \frac{3 \cdot 5 \cdot a}{6} = \log_a \frac{15a}{6}$$

$$= \log_a \boxed{\frac{5a}{2}}$$

Evaluate

$\log_2(\log_3(\cancel{\log_4 64}))$

$\log_2(\cancel{\log_3 3})$

$\log_2(1) = 0$ $\quad 2^0 = 1$

Find the value of x

① $\log_{16} x = -1/4$ $\quad x = 1/2$

$16^{-1/4} = 2^{-1} = 1/2$

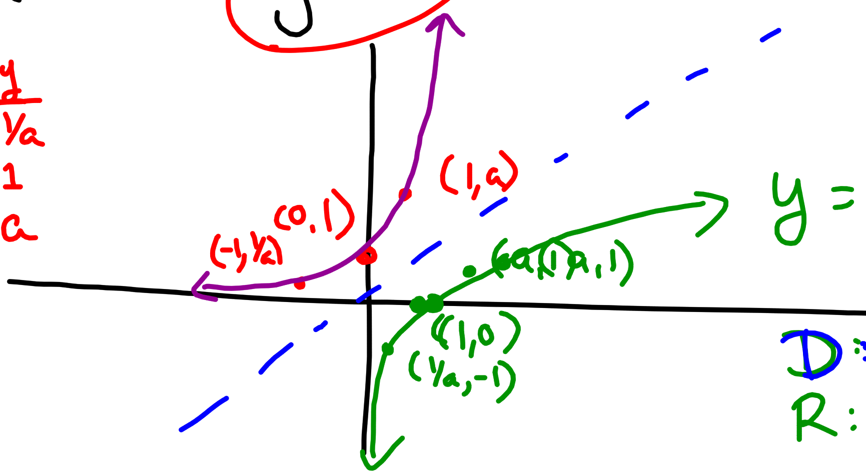
② $\log_x 36 = 2$ $\quad x = 6$

$x^2 = 36$

If $y = a^x$ then $\log_a x = y$
is the <u>INVERSE FUNCTION</u>

Recall: $y = a^x$ $\quad a \neq 1$ and $a > 0$

x	y
-1	$1/a$
0	1
1	a



$y = \log_a x$ always positive

D: $x > 0$

R: $\mathbb{R}$

When graphing $y = \log_a x$ by hand, graph $a^x = y$ <u>FIRST</u> then sketch the inverse.