PRINCIPLES OF MATH 12
LOGARITHM REVIEW

Name: KEY

Except for Questions #5, 9, 21, 22, 23, 24, 26, 28 and 29.

1. Solve for x: $81^{x-1} = \left(\frac{1}{27}\right)^{x-4}$

   A. -8

   B. -3

   C. $-\frac{3}{7}$

   D. $\frac{16}{7}$

$(3^4)^{x-1} = (3^{-3})^{x-4}$

$3^{4x-4} = 3^{-3x+12}$

$4x-4 = -3x+12$

$7x = 16$

$x = 16/7$

2. Solve for x: $9^{x+2} = (3^{4x-3})(3^5)$

   A. 0

   B. 1

   C. $\frac{17}{19}$

   D. $\frac{19}{18}$

$(3^2)^{x+2} = 3^{4x-3+5}$

$3^{2x+4} = 3^{4x+2}$

$2x+4 = 4x+2$

$-2x = -2$

$x = 1$

3. Solve for x: $5 = 3^x$ (Write in logarithmic form)

$\log_3 5 = x$

A. $x = \log_5 3$

B. $x = \log_3 5$

C. $x = 3^5$

D. $x = 5^3$

4. Solve for x: $ab^x = c$ log it!

$\log(ab^x) = \log(c)$

$\log a + \log b^x = \log c$

$\log a + x \log b = \log c$

$-\log a \qquad -\log a$

$\frac{x \log b}{\log b} = \frac{\log c - \log a}{\log b}$

$x = \frac{\log c - \log a}{\log b}$

A. $x = \dfrac{\log c}{\log a + \log b}$

B. $x = \dfrac{\log c + \log a}{\log b}$

C. $x = \dfrac{\log c - \log a}{\log b}$

D. $x = \dfrac{\log c}{\log b} - \log a$

5. Solve algebraically using logarithms: $2^x = 3(5^{x+1})$

(Answer accurate to at least 2 decimal places.)

$\log 2^x = \log 3(5^{x+1})$

$x \log 2 = \log 3 + \log 5^{x+1}$

$x \log 2 = \log 3 + (x+1)\log 5$

$x \log 2 = \log 3 + x \log 5 + \log 5$

$x \log 2 - x \log 5 = \log 3 + \log 5$

$x(\log 2 - \log 5) = \log 3 + \log 5$

$x = \frac{\log 3 + \log 5}{\log 2 - \log 5}$

$x = -2.9554$

6. Solve for x: $\log(3-x)+\log(3+x)=\log 5$

A. $x=-2$

B. $x=2$

C. $x=\pm 2$

D. no solution

$$\log \boxed{(3-x)(3+x)} = \log \boxed{5}$$
$$(3-x)(3+x)=5$$
$$9-x^2=5$$
$$x^2=4$$
$$x=\pm 2$$

7. Solve: $\log_2 8+\log_3 \frac{1}{3}=\log_4 x$

A. $\frac{1}{64}$

B. $\frac{1}{16}$

C. 16

D. 64

$$\log_2 2^3 + \log_3 3^{-1} = \log_4 x$$
$$3 + -1 = \log_4 x$$
$$2 = \log_4 x \quad \text{(un-log it!)}$$
$$4^2 = x$$
$$x = 16$$

8. Solve the following: $\log_2(\log_4(\log_5 x))=-1$

work from the outside in! un-log it one layer at a time.

A. $\frac{1}{25}$

B. 5

C. 25

D. 125

$$2^{-1} = \log_4(\log_5 x)$$
$$\frac{1}{2} = \log_4(\log_5 x)$$
$$4^{1/2} = \log_5 x$$
$$2 = \log_5 x$$
$$5^2 = x$$
$$\underline{x = 25}$$

Check: $\log_5 25 = 2$, $\log_4 2 = \frac{1}{2}$, $\log_2 \frac{1}{2} = -1$ ✓

9. Solve algebraically: $2\log_4 x - \log_4(x+3) = 1$

$\log_4 x^2 - \log_4(x+3) = 1$ 1st combine terms

$\log_4 \frac{(x^2)}{(x+3)} = 1$ Now unlog it

$(x+3)4^1 = \frac{x^2}{\cancel{(x+3)}} \cancel{(x+3)}$

$x^2 = 4x + 12$

$\to x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = -2$ or 6

reject since $\log_4(-2)$ is not possible

$\underline{\underline{x = 6}}$

10. Simplify: $\log_2 4^x$

A. x

B. $2x$

C. 2^x

D. x^2

$\log_2 2^{2x}$

$2x(\underbrace{\log_2 2})$

$2x \cdot 1 = \underline{\underline{2x}}$

11. Write as a single logarithm: $3 + \frac{1}{2}\log_2 x - 3\log_2 y$

Need to write every term as a $\log_2$ of something

A. $\log_2\left(\frac{1000\sqrt{x}}{y^3}\right)$

$\log_2 8 + \log_2 \sqrt{x} - \log_2 y^3$

B. $\log_2\left(\frac{8\sqrt{x}}{y^3}\right)$

$\log_2 \frac{8\sqrt{x}}{y^3}$

C. $\log_2\left(1000 + \sqrt{x} - y^3\right)$

D. $\log_2\left(8 + \sqrt{x} - y^3\right)$

hint: use change of base rule.

12. If $\log_4 x = a$, determine $\log_{16} x$ in terms of a.

$\log_{16} x = \dfrac{\log_4 x}{\log_4 16} = \dfrac{a}{2}$

A. $\frac{a}{4}$

B. $\frac{a}{2}$

C. $2a$

D. $4a$

13. If $\log 2 = a$, $\log 3 = b$, determine an expression for $\log 2400$.

A. $2a^3b$

B. $3a+b+2$

C. $3a+b+100$

D. a^3+b+2

$\log 2400 = \log 24 \cdot 100$

$\log 3 \cdot 8 \cdot 100$

$\log 3 + \log 2^3 + \log 10^2$

$\downarrow \quad 3\log 2 \quad 2\log 10$

$b + 3a + 2.$

14. Simplify: $a^{\log_a 8 + \log_a 2}$

A. 10

B. 16

C. a^{10}

D. a^{16}

$a^{\log_a 8 \cdot 2} = \cancel{\log}\ a^{\log_a 16} = 16.$

15. Determine the value of $\log_n ab^2$ if $\log_n a = 5$ and $\log_n b = 3$.

A. 11
B. 14
C. 16
D. 45

$\log_n ab^2 = \underbrace{\log_n a}_{5} + \underbrace{2\log_n b}_{2(3)}$

$5 + 2(3) = 11$

16. Given $\log_a 2 = x$ and $(\log_a 8)\left(a^{\log_a x}\right) = 12$, solve for a.

A. 2
B. ± 2
C. $\sqrt{2}$
D. $\pm\sqrt{2}$

$\log_a 2^3 \cdot a^{\log_a x} = 12$

$3(x) \cdot x = 12$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2.$

reject $x = -2.$

$x = 2$

17. Change to exponential form: $\log_k \ell = m$

A. $\ell = m^k$
B. $\ell = k^m$
C. $k = m^\ell$
D. $k = \ell^m$

$k^m = \ell$.

18. If (a, b) is on the graph of $y = 3^x$, which point must be on the graph of $y = \log_3 x$?

A. (a, b)
B. (b, a)
C. $(3a, b)$
D. $(a, 3b)$

$y = \log_3 x$ is the INVERSE of $y = 3^x$ so x & y trade places.

19. Determine the inverse of $f(x)=3^{x-1}-2$.

$y=3^{x-1}-2$

$x=3^{y-1}-2$.

$x+2=3^{y-1}$

write in log form.

$\log_3(x+2)=y-1$

$y=\log_3(x+2)+1$

A. $f^{-1}(x)=\log_3(x+2)+1$

B. $f^{-1}(x)=\log_3(x+2)-1$

C. $f^{-1}(x)=\log_3(x-1)+2$

D. $f^{-1}(x)=\log_3(x-1)-2$

20. If \$5000 is invested at 7.2% per annum compounded monthly, which equation can be used to determine the number of years, t, for the investment to increase to \$8000?

$A=P(1+i)^n$ $n=12\cdot t$

$i=\frac{.072}{12}$

A. $8000=5000(1.072)^t$

B. $8000=5000(1.006)^t$

C. $8000=5000(1.072)^{12t}$

D. $8000=5000(1.006)^{12t}$

21. The population of a particular country is 25 million. Assuming the population is growing continuously, the population, P, in millions, t years from now can be determined by the formula $P=25e^{0.022t}$. What will be the population, in millions, 20 years from now?

substitute 20 in for t.

$P=25e^{.022(20)}$ $=38.82$

A. 29.90
B. 37.97
C. 38.63
D. 38.82

22. The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12 000, how many weeks will it take for the population to reach 300 000 ants?

use $A=A_o(m)^{t/\cancel{x}}$

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

$\frac{300000}{12000}=\frac{12000(3)^{t/8}}{12000}$

$25=3^{t/8}$

$\frac{\log 25}{\log 3}=\frac{\frac{t}{8}\log 3}{\log 3}$

$\frac{t}{8}=\frac{\log 25}{\log 3}$

$t=8\left(\frac{\log 25}{\log 3}\right)$

$t=23.4396$ weeks.

so 80% is left

23. The radioactivity of a certain substance decays by 20% in 30 hours. What is the half-life of the substance?

$$.80 = 1\left(\frac{1}{2}\right)^{30/x}$$

$$\log .8 = \log \frac{1}{2}^{30/x}$$

$$x \cdot \log .8 = \cancel{x} \cdot \frac{30}{\cancel{x}} \cdot \log .5$$

$$x = \frac{30 \log .5}{\log .8}$$

$$x = 93.18\underline{\underline{9}}$$

24. The intensity of light reduces by 7% for every 3 metres below the surface of the water. At what depth will the light intensity be reduced to 60% of its original amount?

(60% left).

$$.60 = 1(1-.07)^{x/3}$$

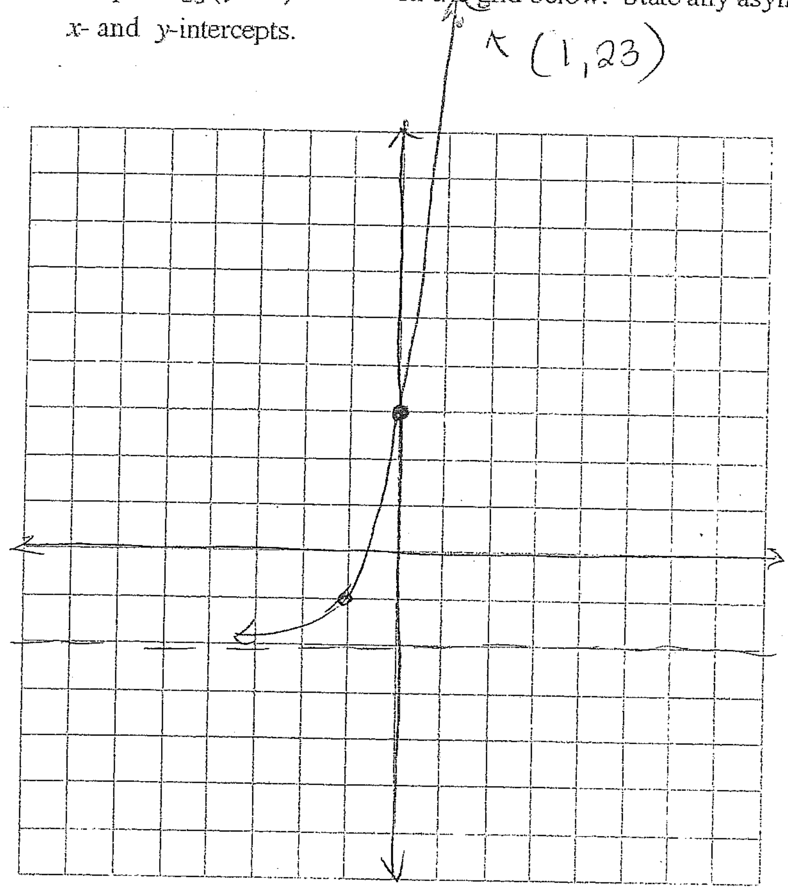
$$.6 = (.93)^{x/3}$$

$$\log .6 = \frac{x}{3} \log .93$$

$$\frac{3 \log .6}{\log .93} = x$$

$$x = 21.12\,m$$

25. Graph $\log_5(y+2) = x+1$ on the grid below. State any asymptotes and determine the x- and y-intercepts.



$y = -2$ is an asymptote.

$\log_5(y+2) = x+1$

is the same as $5^{x+1} = y+2$.

or $y = 5^{x+1} - 2$

which is $y = 5^x$ moved forward one and down 2.

26. The population of Canada is 30 million people and is growing at an annual rate of 1.4%. The population of Germany is 80 million people and is decreasing at an annual rate of 1.7%. In how many years will the population of Canada be equal to the population of Germany?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

Canada : $P = \cancel{P_0}^{30}(1.014)^x$

Germany $P = 80(.983)^x$

want to know when (x) they will be the same

$$\frac{\cancel{30}(1.014)^x}{\cancel{30}} = \frac{80(.983)^x}{30}$$

$$\log(1.014)^x = \cancel{\tfrac{8}{3}}\log\left(\tfrac{8}{3}(.983)^x\right)$$

$$x \log 1.014 = \log \tfrac{8}{3} + x \log .983$$

$$x \log 1.014 - x \log .983 = \log \tfrac{8}{3}$$

$$\frac{x(\log 1.014 - \log .983)}{\log 1.014 - \log .983} = \frac{\log 8/3}{\log 1.014 - \log .983}$$

$\underline{\underline{x = 31.59 \text{ years.}}}$

27. Determine the domain of the function $y = \log(2x+3)$.

A. $x > -\frac{3}{2}$

B. $x > -\frac{2}{3}$

C. $x > \frac{2}{3}$

D. $x > \frac{3}{2}$

Can NOT take the log of a number that is less than or equal to zero, so

$2x+3 > 0$

$x > -3/2$.

28. In 1976, an earthquake in Guatemala had a magnitude of 7.5 on the Richter scale and in 1960, an earthquake in Morocco had a magnitude of 5.8. How many times as intense was the 1976 Guatemalan earthquake compared to the 1960 Moroccan earthquake?

A. 1.29

B. 1.7

C. $10^{1.29}$

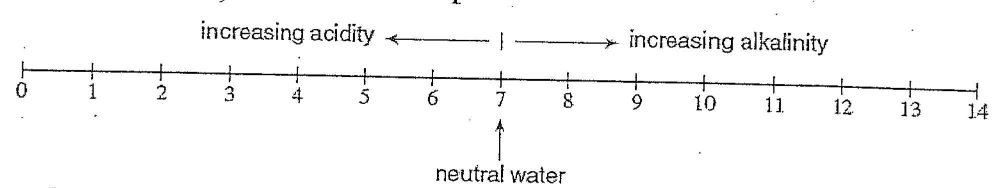
D. $10^{1.7}$

$$\frac{10^{7.5}}{10^{5.8}} = 10^{1.7} = 50.12$$

29. In chemistry, the pH scale measures the acidity (0–7) or alkalinity (7–14) of a solution. It is a logarithmic scale in base 10. Thus, a pH of 5 is 10 times more acidic than a pH of 6.

Solution A has a pH of 5.7. Solution B is 1260 times more acidic than Solution A. Find the pH of solution B.

pH Scale



A. 2.6

B. 4.4

C. 7.0

D. 8.8

$$\frac{10^{5.7}}{10^{x}} = 1260$$

$$\frac{10^{5.7}}{1260} = 10^{x}$$

$$10^{x} = 397.7676$$

$\log_{10} 397.7676 = x$

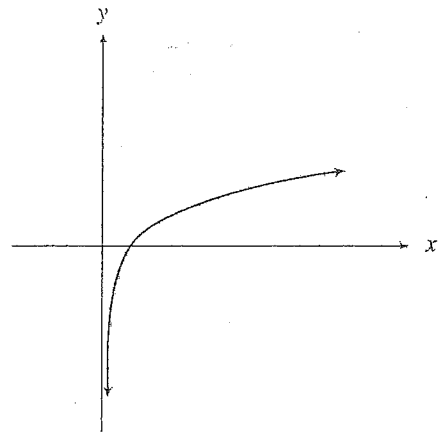
$x = 2.5996$

$x = 2.6$.

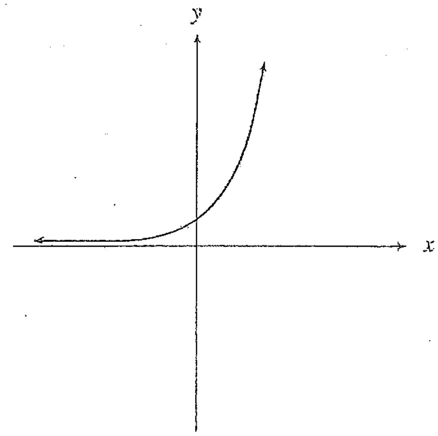
means a is a fraction.

30. If $0 < a < 1$, which of the following is the best graph of $y = \log_a x$?

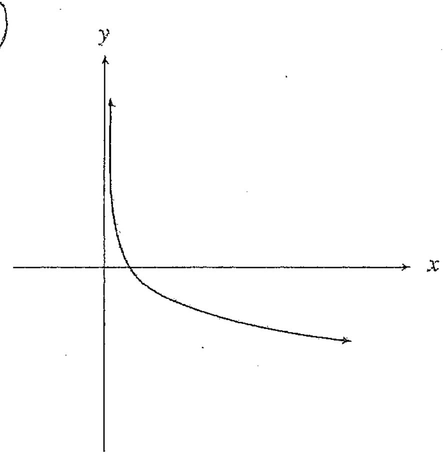
A.



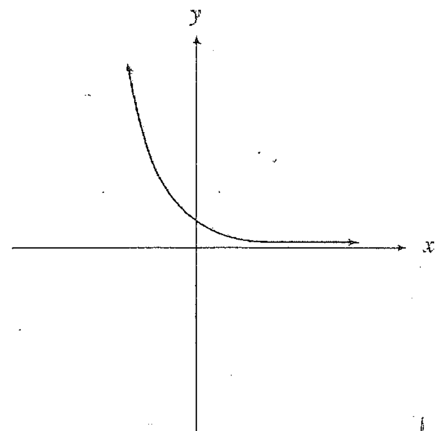
B.



C.



D.

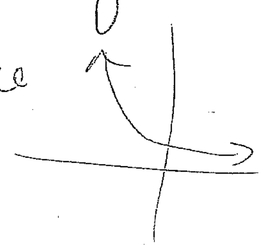


Method I

$y = \log_a x$

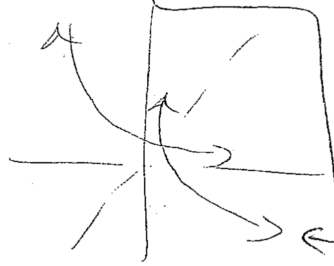
is the inverse of $a^x = y$.

$y = a^x$ looks like



when a is a fraction

so the inverse

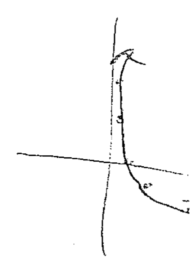


OR Method II

Choose a base:

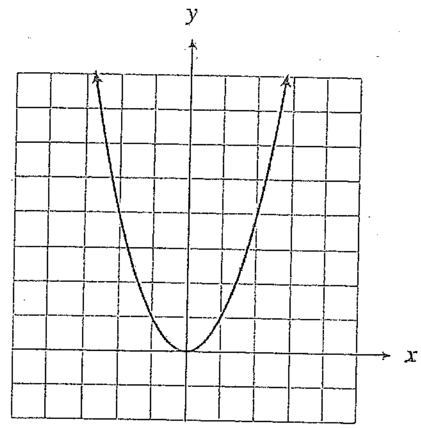
$y = \log_{\frac{1}{2}} x$.

x	y
$\frac{1}{4}$	2
$\frac{1}{8}$	3
1	0
$\frac{1}{2}$	1
2	-1
4	-2

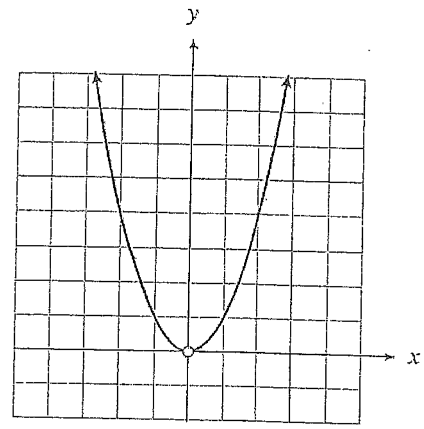


31. Which of the following is a graph of $\log_x y = 2$? $x^2 = y$. BUT see below

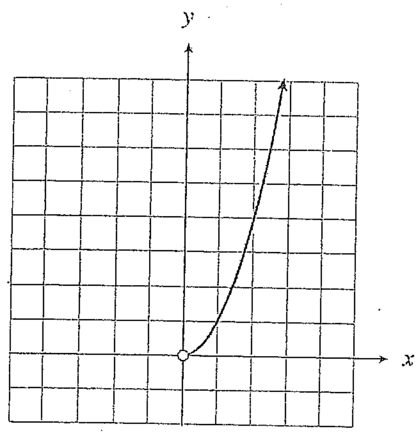
A.



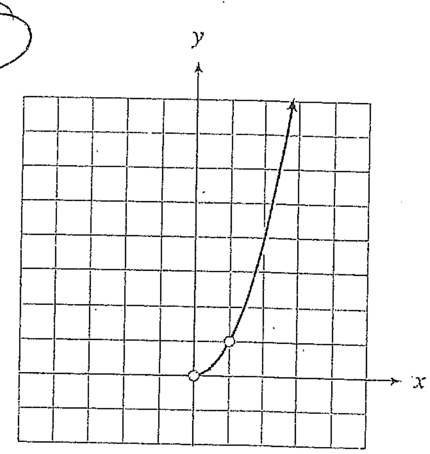
B.



C.



D.



BUT RESTRICTIONS ON LOGARITHMS:

The base must be greater than zero and NOT $= 1$

So $x > 0$ & $x \neq 1$ $\therefore$ Graph D ~~[illegible]~~ is the right equation (parabola) that also includes the appropriate restrictions.

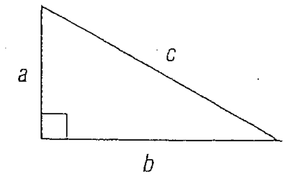
Name ______________________

3.13 The Pythagorean Theorem

MATHPOWER™ [redacted] pp. 108–109

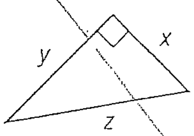
The Pythagorean Theorem states that in any right triangle, if c is the length of the hypotenuse, and a and b are the lengths of the legs, then

$$a^2 + b^2 = c^2$$



State the relationship in the form $a^2 + b^2 = c^2$ for the sides in each triangle.

1.



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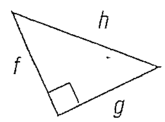
2.



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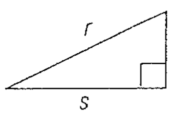
3.



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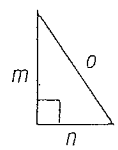
4.



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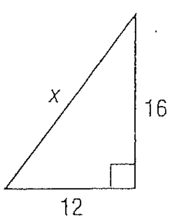
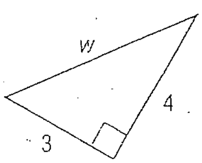
Find the length of the unknown side in each right triangle.

5.



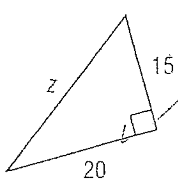
$x =$ ______________

6.



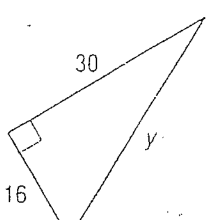
$w =$ ______________

7.



$z =$ ______________

8.

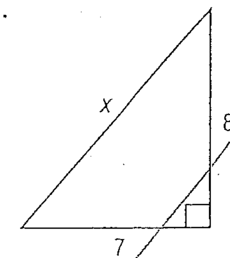


$y =$ ______________

IGNORE!

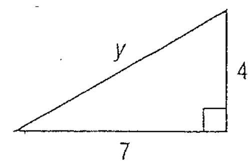
Calculate the length of the unknown side, to the nearest tenth.

9.



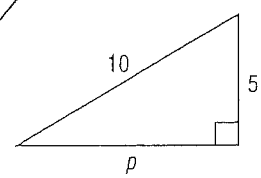
$x =$ ______________

10.



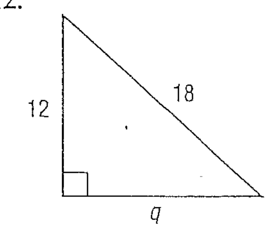
$y =$ ______________

11.



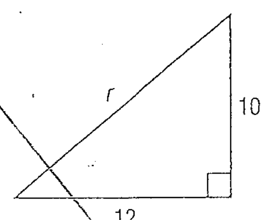
$p =$ ______________

12.



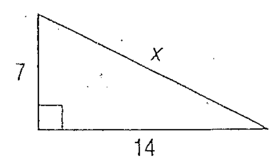
$q =$ ______________

13.



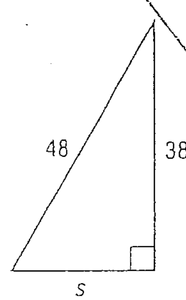
$r =$ ______________

14.



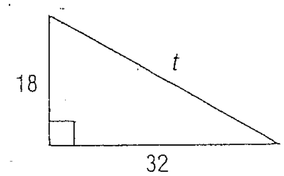
$x =$ ______________

15.



$s =$ ______________

16.



$t =$ ______________