Permutations Involving Distinct Objects

Ex 1 How many ways can you arrange:

(a) the letters A and B?

Permutation: AB or BA — 2 — $2 \cdot 1 = 2$

(b) the letters A, B and C?

Permutations:

ABC CBA
BAC BCA
ACB CAB

6 — $\underline{3} \cdot \underline{2} \cdot \underline{1}$

(c) the letters A B C and D?

6	6	6	6
ABCD	BACD		
ABDC	BADC		
ACBD	BCAD	...	
ACDB	BCDA		
ADBC	BDAC		
ADCB	BDCA		

$4 \cdot 6 = \underline{24}$

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

(D) 7 letters:

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \underline{5040}$

$12 \cdot 11 \cdot 10 \cdot \ldots \cdot 1 = \_\_\_$

$12! \rightarrow$ 12 "factorial"

→ means $12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

! is a mathematical symbol called a "factorial"

$(-4)! = -4 \cdot -5 \cdot -6 \cdot -7 \ldots$ huh?!!

You can't take the factorial of a negative #!!

→ also only works for whole #'s

By Definition $\boxed{0! = 1}$

7 math → → → 4 = $7! = 5040$

A PERMUTATION is an arrangement of objects in which order is important.

How many ways can two letters be chosen from ABCD and E?

20

$\underline{5} \cdot \underline{4} = 20$

AB BA CA
AC BC CB
AD BD CD
AE BE CE ...

5 columns, 4 in each
$5 \times 4 = 20$

6 letters, want permutations of 2?

$\underline{6} \cdot \underline{5} = 30$

6 letters, wanted permutations of 4?

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 360$

$$\frac{(6 \cdot 5 \cdot 4 \cdot 3) \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6!}{2!} = \frac{6!}{(6-4)!}$$

If we have 10 distinct objects and we want to arrange them in permutations of 3, we write

$$P = \frac{!}{(\;)!} = \frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$= 10 \cdot 9 \cdot 8 = 720$

$_nP_r$

(on calculator)

$_{10}P_3 = 10 \; {}_nP_r \; 3 = 720$

$$_nP_r = \frac{n!}{(n-r)!}$$

Now, let's make it fun:

① ABC taken in permutations of 3:

ABC
ACB
BAC
BCA
CAB
CBA

$\underline{\underline{6}}$

Which means:

$${}_3P_3 = \frac{3!}{(3-3)!} = 6$$

$$\downarrow$$

$$\frac{3!}{0!} = 6$$

$$\downarrow$$

$$\frac{3\cdot 2\cdot 1}{0!} = 6$$

$0! \Rightarrow$ must equal 1

$$\cancel{y}\cdot \frac{3!}{\cancel{y}} = 6\cdot y$$

$$3! = 6y$$

$$\downarrow$$

$$\frac{6}{6} = \frac{6y}{6}$$

$y=1$ so $0!=1$

Write with $\underline{\underline{\text{OUT}}}$ a ! symbol:

$${}_nP_2 = \frac{n!}{(n-2)!} = \frac{n\cdot(n-1)\cdot\cancel{(\ \ )}\cdot\cancel{(\ \ )!}}{\cancel{(\ \ )}\cdot\cancel{(\ \ )!}}$$

$$= n(n-1)$$

$$= n^2-n$$

$$\frac{(n+2)!}{(n-1)!} = \frac{(n+2)\cdot(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$$

$$= (n+2)(n+1)(n) = n^3+3n^2+2n$$

${}_nP_2 = 42$ find $\underline{n}$

$$\frac{n!}{(n-2)!} = 42$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$

${}_7P_2 \overset{?}{=} 42$

$n(n-1) = 42$ so $\underline{\underline{n=7}}$

$n^2-n-42=0$

$(n-7)(n+6)=0$

so $n=7$ or $\cancel{n=-6}$ reject

${}_{10}P_n = 90$ means $\dfrac{10!}{(10-n)!} = 90$

$$10! = 90(10-n)!$$

$$\downarrow$$

$$\frac{\boxed{\cancel{10\cdot 9}}\cdot 8!}{\cancel{90}} = \frac{\cancel{90}(10-n)!}{\cancel{90}}$$

$$8! = (10-n)!$$

$$\downarrow$$

must be 8

so $10-n=8$

$\therefore n=2$