2.4 The Product Rule

The sum & difference rule work so well ... $(f+g)' = f' + g'$, will the same idea work for the product $f \cdot g$?

i.e. $\underbrace{((x^3)(x^2))'}_{(x^5)'} \stackrel{?}{=} (x^3)' \cdot (x^2)'$

$5x^4 \quad = 3x^2 \cdot 2x$

$\neq 6x^3$ NOPE

In general, $\underbrace{(f \cdot g)'(x)}_{(f(x) \cdot g(x))'} \neq f'(x) \cdot g'(x)$

Instead:

$$(f \cdot g)' = f \cdot g' + f' \cdot g$$

$f(x) = x^3$

$g(x) = x^2$

$(f \cdot g)' = (x^3)(2x) + (3x^2)(x^2)$

$= 2x^4 + 3x^4 = 5x^4$

Ex 2 $\quad y = (\overset{f}{2x^2+7})(\overset{g}{7x^5+9})$

$$\frac{dy}{dx} = y' = \overset{f}{(2x^2+7)} \cdot \overset{g'}{(35x^4)} + \overset{f'}{(4x)} \cdot \overset{g}{(7x^5+9)}$$

$$= 70x^6 + 245x^4 + 28x^6 + 36x$$

$$= \underline{\underline{98x^6 + 245x^4 + 36x}}$$

2.5 The Quotient Rule

If $F(x) = \dfrac{f(x)}{g(x)}$

Then $F'(x) = \dfrac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

Example:

$$F(x) = \frac{\boxed{x^2+2x-3}}{\boxed{x^3+1}} \quad \begin{matrix} \to f(x) \\ \to g(x) \end{matrix}$$

$$F'(x) = \frac{\overset{g}{(x^3+1)} \cdot \overset{f'}{(2x+2)} - \overset{f}{(x^2+2x-3)} \cdot \overset{g'}{(3x^2)}}{\underset{g^2}{(x^3+1)^2}}$$

$$F'(x) = \frac{2x^4+2x^3+2x+2 - (3x^4+6x^3-9x^2)}{(x^3+1)^2}$$

$$= \frac{-x^4-4x^3+9x^2+2x+2}{(x^3+1)^2}$$

Now: You can work on everything up to the end of Sect. 2.5 ☺