Quadratic Equations

① By Completing the Square

⇒ Works if you need to graph the equation as well
⇒ Uses algebra + not a formula

Ex: $(x^2-6x)-2=0$

$(x^2-6x+9)-9-2=0$

$\frac{6}{2}=3 \Rightarrow 3^2$

$(x-3)^2-11=0$

$+11 \quad +11$

$\sqrt{(x-3)^2}=\sqrt{11}$

$x-3=\pm\sqrt{11}$

$+3 \quad +3$

$x=3\pm\sqrt{11}$

(2) Solve by factoring

- Quick & easy IF the equation can be factored!!

⇒ $x^2-6x-2=0$ Can't factor

$x^2-7x+12=0$

$(x-3)(x-4)=0$ so $x=3$ or $x=4$

Roots / answers / solutions / zeros

(3) Solve by graphing:

⇒ by hand: complete the square & put it in vertex form

PROS ⇒ Easy to see answers ⇒ x-intercepts

⇒ Easy to do on a GC

⇒ Easy to see how many solutions there are

CONS ⇒ hard to draw accurate graphs by hand!

⇒ If it has weird answers, window can be hard to find on GC

(4) THE QUADRATIC FORMULA

If $ax^2+bx+c=0$ · Always works!!

then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Ex: $x^2-2x-3=0$

$a=1$, $b=-2$, $c=-3$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-3)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm \sqrt{16}}{2}$$

$$= \frac{2+4}{2} \quad \text{or} \quad \frac{2-4}{2}$$

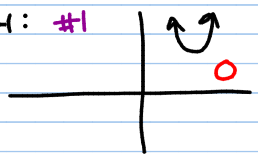
$$\downarrow$$

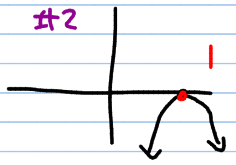
$$x = 3 \quad \text{or} \quad -1$$

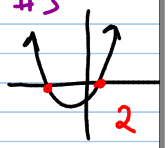
Number of Solutions:

A quadratic Eqn can have:

0, 1 or 2

GRAPH: #1 

#2 

#3 

- If it is factorable, it has 2 solutions
- Solving by completing the square:
  - $\sqrt{(x+2)^2} = \sqrt{-9}$ STOP — No solutions (Graph #1)
  - $\sqrt{(x+2)^2} = \sqrt{0}$ — 1 solution (Graph #2)

    $x + 2 = \pm 0$
  - $\sqrt{(x+2)^2} = \sqrt{16}$ — 2 solutions (Graph #3)

    $x + 2 = \pm 4$

Quad form

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

→ b^2-4ac is called the "Discriminant"

If $b^2-4ac < 0$ → No solutions!!

If $b^2-4ac = 0$ → 1 solution

If $b^2-4ac > 0$ → 2 solutions

Examples:

① $x^2 - 6x + 2 = 0$

Discriminant: (b^2 − $4ac$)

$(-6)^2 - 4(1)(2)$

$36 - 8 = 28$

so there will be 2 solutions since $b^2-4ac > 0$

② $x^2 + 10x + 50 = 0$

$(10)^2 - 4(1)(50)$

$100 - 200 = -100$

No solutions (0 solutions)

Solve these using the QF

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(1) $-2x^2 + 3x = -8$

$-2x^2 + 3x + 8 = 0$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(-2)(8)}}{2(-2)}$

$x = \frac{-3 \pm \sqrt{9 + 64}}{-4}$

$x = \frac{-3 \pm \sqrt{73}}{-4}$

so $x = -1.386$ ←

or $x = 2.886$ ←

(2) $3x^2 - 5x + 9 = 0$

$x = \frac{5 \pm \sqrt{25 - 4(3)(9)}}{2(3)}$

$x = \frac{5 \pm \sqrt{-83}}{6}$ No solution

(3) $\frac{1}{4}x^2 - 3x + 9 = 0$

$x = \frac{3 \pm \sqrt{(9) - 4(\frac{1}{4})(9)}}{2(1/4)}$

$x = \frac{3 \pm \sqrt{9 - 9}}{2(1/4)} = \frac{6}{}$

4.4

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