Pg 92 Section 2.4 $(fg)' = fg' + f'g$

1. a) $f'(x) = (2x-1)(2x) + (2)(x^2+1)$

c) $y' = x^2(1-6x) + 2x(1+x-3x^2)$

e) $f'(t) = (t^4+t^2-1)(2t) + (4t^3+2t)(t^2-2)$

g) $F'(y) = \sqrt{y}\,(1 - \cancel{2}y^{-1/2}) + \frac{1}{2}y^{-1/2}(y - 2\sqrt{y} + 2)$

OR

$= \sqrt{y}\left(1 - \frac{1}{\sqrt{y}}\right) + \frac{1}{2\sqrt{y}}(y - 2\sqrt{y} + 2)$

2. a) $y' = x^3(2x+2) + 3x^2(x^2+2x+3)$

$= 2x^4 + 2x^3 + 3x^4 + 6x^3 + 9x^2$

$= 5x^4 + 8x^3 + 9x^2$

c) $f'(x) = (1-x^2)(-3x^2) + (-2x)(2-x^3)$

$= -3x^2 + 3x^4 - 4x + 2x^4$

$= 5x^4 - 3x^2 - 4x$

e) $f'(t) = (6+t^{-2})(80t^9 - 15t^2) + (-2t^{-3})(8t^{10} - 5t^3)$

$= 480t^9 - 90t^2 + 80t^7 - 15 - 16t^7 + 10$

$= 480t^9 + 64t^7 - 90t^2 - 5$

g) $g'(u) = \sqrt{u}\,(-2u + 20u^3) + \frac{1}{2}u^{-1/2}(2 - u^2 + 5u^4)$

$= -2u^{3/2} + \overset{10/2}{20}u^{7/2} + u^{-1/2} - \frac{1}{2}u^{3/2} + \frac{5}{2}u^{7/2}$

$= \frac{45}{2}u^{7/2} - \frac{5}{2}u^{3/2} + u^{-1/2}$

3\. a) $y' = (1-2x)(3) + (-2)(3x-4)$

$= 3 - 6x - 6x + 8$

$= -12x + 11 \quad \text{at } x = 2$

$m = -12(2) + 11$

$m = -13$

c) $y' = x^4(12x^2) + 4x^3(4x^3+2)$

$= 12x^6 + 16x^6 + 8x^3$

$= 28x^6 + 8x^3 \quad \text{at } x = -1$

OOPS!! Answer in text is wrong (?)

$m = 28(-1)^6 + 8(-1)^3$

$= \underline{\underline{20}}$

e) $y' = x^{-5}(-x^{-2}) + -5x^{-6}(1+x^{-1})$

$= -x^{-7} - 5x^{-6} - 5x^{-7}$

$= -6x^{-7} - 5x^{-6} \quad \text{at } x = 1$

$= -6 - 5$

$= -11$

4\. a) $f'(x) = (6x^4 - 3x^2 + 1)(-3x^2) + (24x^3 - 6x)(2-x$

$= -18x^6 + 9x^4 - 3x^2 + 48x^3 - 24x^6 - 12x + 6x^4$

$= -42x^6 + 15x^4 + 48x^3 - 3x^2 - 12x$

$f'(1) = -42 + 15 + 48 - 3 - 12 = 6$

b) $f(x) = (6x^4 - 3x^2 + 1)(2 - x^3)$

$= 12x^4 - 6x^2 + 2 - 6x^7 + 3x^5 - x^3$

$= 48x^3 - 12x - 42x^6 + 15x^4 - 3x^2$

$= -42x^6 + 15x^4 + 48x^3 - 3x^2 - 12x$

$f'(1) = 6$

5. $y' = (2-\sqrt{x})\left(\frac{1}{2}x^{-1/2}+3\right) + \left(-\frac{1}{2}x^{-1/2}\right)\left(1+x^{1/2}+3x\right)$

$y'(1) = (2-1)\left(\frac{1}{2}+3\right) + (-1/2)(1+1+3)$

$= \frac{7}{2} - \frac{5}{2} = 1$

$y(1) = (2-1)(1+1+3)$

$= 5.$

$y - 5 = x - 1$

$\boxed{x - y + 4 = 0}$

6. $f(2)=3 \quad f'(2)=5 \quad g(2)=-1 \quad g'(2)=-4$

$(fg)'(2) = f\,g' + f'g$

$= 3\cdot -4 + 5\cdot(-1)$

$= -12 - 5$

$= \underline{\underline{-17}}$

7. a) $g'(x) = x\cdot f'(x) + f(x).$

b) $h'(x) = \cancel{\frac{1}{2\sqrt{\,}}}\sqrt{x}\,f'(x) + \frac{1}{2\sqrt{x}}f(x)$

c) $F'(x) = x^c f'(x) + cx^{c-1}f(x)$

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$$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

1. a) $$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$$

$$= \frac{2}{(x+1)^2}$$

c) $$g'(x) = \frac{(x^2+2x-1) - (x)(2x+2)}{(x^2+2x-1)^2}$$

$$= \frac{-x^2 \cancel{} - 1}{(x^2+2x-1)^2}$$

e) $$y' = \frac{(x^2+1)(\frac{1}{2}x^{-1/2}) - \sqrt{x}(2x)}{(x^2+1)^2}$$

$$= \frac{\frac{1}{2}x^{3/2} + \frac{1}{2}x^{-1/2} - 2x^{3/2}}{(x^2+1)^2}$$

$$= \frac{-\frac{3}{2}x^{3/2} + \frac{1}{2}x^{-1/2}}{(x^2+1)^2} = \frac{\frac{-3x^{3/2}\sqrt{x}}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}}{(x^2+1)^2}$$

$$= \frac{-3x^2+1}{2\sqrt{x}(x^2+1)^2}$$

$$\frac{\frac{1}{2}}{3} = \frac{1}{2} \times \frac{1}{3}$$

$$\frac{1}{2}x^{\frac{1}{3}}$$

j) $$f'(t) = \frac{(t^2-3t+4)(2) - (2t+1)(2t-3)}{(t^2-3t+4)^2}$$

$$= \frac{2t^2-6t+8-4t^2+4t+3}{(t^2-3t+4)^2}$$

$$= \frac{-2t^2-2t+11}{(t^2-3t+4)^2}$$

i) $$f'(x) = \frac{(x^4-x^2+1)(0) - (1)(4x^3-2x)}{(x^4-x^2+1)^2}$$

$$= \frac{-4x^3+2x}{(x^4-x^2+1)^2}$$

k) $$f'(x) = \frac{(x^5-10)(6x^5) - (x^6)(5x^4)}{(x^5-10)^2}$$

$$= \frac{6x^{10}-60x^5-5x^{10}}{(x^5-10)^2}$$

$$= \frac{x^{10}-60x^5}{(x^5-10)^2}$$

2 a) $f(x) : x \neq \frac{1}{2}$

$$f'(x) = \frac{(1-2x)(1) - \overbrace{(-2)(2+x)}^{-4-2x}}{(1-2x)^2}$$

$$= \frac{1-2x+4+2x}{(1-2x)^2} = \frac{5}{(1-2x)^2}$$

c) $f(x) = \dfrac{1}{(2x^2-x-3)}$ $\quad x \neq -1, \frac{3}{2}$

$$f'(x) = \frac{0-(4x-1)}{(2x^2-x-3)^2}$$

$$= \frac{-4x+1}{(2x^2-x-3)^2}$$

e) $f'(x) = \dfrac{(x^4-1)(2x+2) - (4x^3)(x^2+2x)}{(x^4-1)^2}$

$x \neq \pm 1$

$$= \frac{2x^5+2x^4-2x-2-4x^5-8x^4}{(x^4-1)^2}$$

$$= \frac{-2x^5-6x^4-2x-2}{(x^4-1)^2}$$

3. a) $y = \frac{x}{x-2}$ $(4, 2)$

$$y' = \frac{(x-2)(1) - (1)(x)}{(x-2)^2}$$

$$= \frac{x - 2 - x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$$y'(4) = \frac{-2}{(4-2)^2} = \frac{-2}{4} = -\frac{1}{2}$$

$$y - 2 = -\frac{1}{2}(x - 4)$$

$$2y - 4 = -x + 4$$

$$\boxed{x + 2y - 8 = 0}$$

c) $y' = \frac{0 - (1)(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$

$$y'(-2) = \frac{-2(-2)}{(4+1)^2} = \frac{+4}{25}$$

$$y - \frac{1}{5} = \frac{+4}{25}(x + 2)$$

$$25y - 5 = +4x + 8$$

$$\boxed{4x + 25y + 13 = 0}$$

Thank you, Corinne!

4. $\left(\frac{f}{g}\right)'(2) = \frac{g(2)(f'(2)) - g'(2)(f(2))}{g(2)^2}$

$= \frac{(-1)(5) - (-4)(3)}{(-1)^2} = -5 + 12 = \underline{7}$

5. $y' = \frac{(3x+4)(1) - (x+2)(3)}{(3x+4)^2}$

$= \frac{3x+4-3x-6}{(3x+4)^2} = \frac{-2}{(3x+4)^2}$

$(3x+4)^2$ is allways positive so $\frac{-2}{(3x+4)^2}$ will always be negative.

6. $y' = \frac{(2x+5)(2x) - (x^2)(2)}{(2x+5)^2}$

$= \frac{4x^2+10x-2x^2}{(2x+5)^2} = \frac{2x^2+10x}{(2x+5)^2}$ $\left(x \neq -\frac{5}{2}\right)$

y' will be zero when the numerator $= 0$

$2x^2 + 10x = 0$

$2x(x+5) = 0$ when $x = 0$ or -5

When $x = 0$

$y = \frac{0}{0+5} = 0$ $(0,0)$

When $x = -5$ $(-5,-5)$

$y = \frac{(25)}{-5} = -5$

6. 7. $y = -\frac{1}{4}x + \frac{1}{4}$ want $m = -\frac{1}{4}$

$$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2}$$

$\frac{-1}{(x-1)^2} = \frac{-1}{4}$ when $x = -1$ or 3

When $x = -1$: $y = \frac{-1}{-1-1}$

$\cancel{y = -\frac{1}{4}(-1) + \frac{1}{4}} = \frac{1}{2}$ $(-1, \frac{1}{2})$

$x = 3$

$\cancel{y = -\frac{1}{4}(3) + \frac{1}{4}} = -\frac{1}{2}$ $(3, \frac{3}{2})$

$y = \frac{3}{3-1} = \frac{3}{2}$

$4 = (x-1)^2$