2.6 The Chain Rule #1-10

1. a) $F'(x) = 7(5-3x)^6(-3)$
$= -21(5-3x)^6$

c) $G'(x) = \frac{3}{4}(x^3+x^2-2)^{-1/4} \cdot (3x^2+2x)$

$= \frac{3(3x^2+2x)}{4(x^3+x^2-2)^{1/4}}$

e) $y' = \frac{1}{4}(x^2+x)^{-3/4}(2x+1)$

$= \frac{2x+1}{4\sqrt[4]{(x^2+x)^3}}$ OR $\frac{2x+1}{4(x^2+x)^{3/4}}$

g) $y = (x^3+2x^2+1)^{-2}$

$y' = -2(x^3+2x^2+1)^{-3}(3x^2+4x)$
$= \frac{-2(3x^2+4x)}{(x^3+2x+1)^3}$

OR

$= \frac{-6x^2-8x}{(x^3+2x+1)^3}$

i) $y = (1+2\sqrt{x})^6$

$y' = 6(1+2\sqrt{x})^5 \cdot \left(\frac{1}{\sqrt{x}}\right)$

$= \frac{6(1+2\sqrt{x})^5}{\sqrt{x}}$

12) $y' = 1 - \frac{1}{5}(1+x^5-6x^{10})^{-4/5}(5x^4-60x^9)$

$$= 1 - \frac{(\cancel{5}x^4 - \overset{12}{\cancel{60}}x^9)}{\cancel{5}(1+x^5-6x^{10})^{4/5}}$$

$$= 1 - \frac{x^4 - 12x^9}{(1+x^5-6x^{10})^{4/5}}$$

2. $\frac{dy}{dx} = (4u^3 + 10u)(5x^4 + 4x)$

3. $\frac{dy}{dx}\Big]_{x=4} = (2u - 10u^4)(1 - \frac{1}{2}x^{-1/2})$

$$= [2(x - \sqrt{x}) - 10[x - \sqrt{x}]^4)(1 - \frac{1}{2\sqrt{x}})$$

$$= [2(4 - \overset{2}{2}) - 10[4 - \overset{2,4}{2}]^4)(1 - \frac{1}{4})$$

$$= [4 - \overset{160}{\cancel{2560}}]\left[\frac{3}{4}\right]$$

$$= -156 \cdot \frac{3}{4} = -117$$

4. $\frac{dy}{dt}\Big]_{t=1}$ if $y = \sqrt{1 + r^2}$ and $r = \frac{t+1}{2t+1}$ <u>oops</u>

$$\frac{dy}{dt} = \frac{1}{2}(1+r^2)^{-1/2}\left(\frac{(2t+1)(1) - (t+1)(2)}{(2t+1)^2}\right)$$

$$= \frac{1}{2}\left(1 + \frac{(t+1)^2}{(2t+1)^2}\right)^{-1/2}\left(\frac{-1}{(2t+1)^2}\right)$$

at $t=1$ $y'(1) = \frac{1}{2}\Big[\underbrace{1 + \frac{4}{9}}_{\frac{13}{9}}\Big]^{-1/2}\left[\frac{-1}{9}\right] = \frac{1}{2} \cdot \frac{\cancel{3}}{\sqrt{13}} \cdot \frac{-1}{\cancel{9}_3}$

$$= \frac{-1}{6\sqrt{13}}$$

#4 $\left.\dfrac{dy}{dt}\right]_{t=1}$ if $y=\sqrt{1+r^2}$ $\quad r=\dfrac{t+1}{2t+1}$

$u=1+r^2$

$$y'=\frac{1}{2}(1+r^2)^{-1/2}\cdot\frac{du}{dt}$$

$$\frac{du}{dt}=\frac{d}{dt}(1+r^2)$$

$$=0+2r\frac{dr}{dt}$$

$$=2r\left[\frac{(2t+1)(1)-(t+1)(2)}{(2t+1)^2}\right]$$

$$=2\left[\frac{t+1}{2t+1}\right]\left[\frac{-1}{(2t+1)^2}\right]=\frac{-2(t+1)}{(2t+1)^3}$$

so: $$\frac{dy}{dt}=\frac{1}{2}\left[1+\frac{(t+1)^2}{(2t+1)^2}\right]^{-1/2}\cdot\frac{-2(t+1)}{(2t+1)^3}$$

$$y'(1)=\frac{1}{2}\left[1+\frac{4}{9}\right]^{-1/2}\cdot\frac{-4}{27}$$

$$=\frac{1}{2}\left[\frac{13}{9}\right]^{-1/2}\cdot\frac{-4}{27}$$

$$=\frac{1}{\cancel{2}_{1}}\cdot\frac{\cancel{3}^{1}}{\sqrt{13}}\cdot\frac{-\cancel{4}^{2}}{\cancel{27}_{9}}=\boxed{\frac{-2}{9\sqrt{13}}}$$

5. $\left.\frac{ds}{dt}\right]_{t=4}$ if $s = v + \frac{50}{v}$ and $v = 3t - \sqrt{t}$

$\frac{dv}{dt} = 3 - \frac{1}{2}t^{-1/2}$

$$s' = 3 - \frac{1}{2}t^{-1/2} + (-50)v^{-2} \cdot \frac{dv}{dt}$$

$$= 3 - \frac{1}{2}t^{-1/2} - \frac{50}{(3t-\sqrt{t})^2} \cdot \left(3 - \frac{1}{2}t^{-1/2}\right)$$

$$s'(4) = 3 - \frac{1}{2\sqrt{4}} - \frac{50}{(12-2)^2}\left(3 - \frac{1}{2\sqrt{4}}\right)$$

$$= 3 - \frac{1}{4} - \frac{1}{2}\left(3 - \frac{1}{4}\right)$$

$$= \frac{11}{4} - \frac{1}{2}\left(\frac{11}{4}\right) =$$

$$= \frac{22}{8} - \frac{11}{8} \boxed{= \frac{11}{8}}$$

6. (a) $F'(x) = x\sqrt{x^2+1}$ -use product rule and chain rule

$$= x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + (1)\left(\sqrt{x^2+1}\right)$$

$$= \frac{x^2}{\sqrt{x^2+1}} + \sqrt{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$ common denom

$$= \frac{x^2+x^2+1}{\sqrt{x^2+1}} = \boxed{\frac{2x^2+1}{\sqrt{x^2+1}}}$$

(c) $G(x) = (x^2-1)^4(2-3x)$

$= (x^2-1)^4(-3) + (2-3x)4(x^2-1)^3(2x)$

$= -3(x^2-1)^4 + 8x(2-3x)(x^2-1)^3$

$= (x^2-1)^3[-3(x^2-1) + 8x(2-3x)]$

$= (x^2-1)^3[-3x^2+3+16x-24x^2]$

$= (x^2-1)^3[-27x^2+16x+3]$

(e) $F(x) = \dfrac{x}{\sqrt{2x+3}}$

$$F'(x) = \frac{(2x+3)^{1/2}(1) - x\,\frac{1}{2}(2x+3)^{-1/2}(2)}{((2x+3)^{1/2})^2}$$

$$= \frac{\frac{(2x+3)^{1/2}(2x+3)^{1/2}}{(2x+3)^{1/2}} - \frac{x}{(2x+3)^{1/2}}}{2x+3}$$

$$= \frac{\frac{2x+3-x}{(2x+3)^{1/2}}}{(2x+3)}$$

$$= \frac{x+3}{(2x+3)^{3/2}}$$

(g) $g(x) = \left(\dfrac{x+2}{x-2}\right)^3$

$x-2-x-2$

$$g'(x) = 3\left(\frac{x+2}{x-2}\right)^{2}\left[\frac{(x-2)(1)-(x+2)(1)}{(x-2)^2}\right]$$

$$= \frac{3(x+2)^2(-4)}{(x-2)^4} = \frac{-12(x+2)^2}{(x-2)^4}$$

(i) $y = \sqrt{\dfrac{x^2-1}{x^2+1}}$

$2x^3+2x \quad -2x^3+2x$

$$y' = \frac{1}{2}\left(\frac{x^2-1}{x^2+1}\right)^{-1/2}\left[\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}\right]$$

$$= \frac{1}{\cancel{2}}\left(\frac{x^2+1}{x^2-1}\right)^{1/2}\left[\frac{\cancel{2}4x}{(x^2+1)^2}\right]$$

$$= \frac{2x\sqrt{x^2+1}}{\sqrt{(x^2-1)}(x^2+1)^2} = \frac{2x(x^2+1)^{1/2}}{(\sqrt{x^2-1})(x^2+1)^2}$$

$$= \frac{2x}{(\sqrt{x^2-1})(x^2+1)^{3/2}}$$

(k) $y = 3\sqrt{x}(2x+1)^5 + \sqrt{4x-3}$

$$= 3\sqrt{x}\cdot 5(2x+1)^4(2) + 3\cdot\frac{1}{2}x^{-1/2}(2x+1)^5 + \frac{1}{2}(4x-3)^{-1/2}(4)$$

$$= 30\sqrt{x}(2x+1)^4 + \frac{3(2x+1)^5}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= 30\sqrt{x}(2x+1)^4\cdot\frac{2\sqrt{x}}{2\sqrt{x}} + \frac{3(2x+1)^5}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= \frac{60x(2x+1)^4 + 3(2x+1)^5}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

PTO

$$= \frac{(2x+1)^4[60x+3(2x+1)]}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= \frac{(2x+1)^4[60x+6x+3]}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= \frac{(2x+1)^4[66x+3]}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= \frac{(2x+1)^4 \cdot 3(22x+1)}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

$$= \frac{3(2x+1)^4(22x+1)}{2\sqrt{x}} + \frac{2}{\sqrt{4x-3}}$$

l) $y = \sqrt{1+\sqrt[3]{x}}$

$$y' = \frac{1}{2}\left(1+\sqrt[3]{x}\right)^{-1/2}\left(\frac{1}{3}x^{-2/3}\right)$$

$$= \frac{1}{6(1+\sqrt[3]{x})^{1/2}(x^{2/3})}$$

n) $y = \sqrt{x+\sqrt{x+\sqrt{x}}}$

$$y' = \frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left[1+\frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$$

OR $y' = \left(\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\right)\left(\left[1+\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}\right]\right)$

7. $y = (x^2-3)^8$ at $(2,1)$

$y' = 8(x^2-3)^7(2x)$

$y'(2) = 8(4-3)^7(4)$
$= 32$

$y-1 = 32(x-2)$

$\boxed{32x - y - 63 = 0}$

8. $y = \dfrac{1}{\sqrt{20-x^4}}$ at $(2, 1/2)$

$y' = -\frac{1}{2}(\cancel{\sqrt{20-x^4}})^{-3/2}(-\overset{2}{\cancel{4}}x^3)$

$= \dfrac{2x^3}{(20-x^4)^{3/2}}$

$y'(2) = \dfrac{2(2^3)}{(20-16)^{3/2}} = \dfrac{16}{(\sqrt{4})^3} = \dfrac{16}{8} = 2$

$y - \frac{1}{2} = 2(x-2)$

$2y - 1 = 4(x-2)$

$\boxed{4x - 2y - 7 = 0}$

9. $g(2)=4 \quad g'(2)=3 \quad f'(4)=5.$

$F(x)=f(g(x))$

$F'(x)=f'(g(x))\cdot g'(x)$

$F'(2)=f'(g(2))\cdot g'(2)$

$=[f'(4)](3)$

$=5\cdot 3=\boxed{15}$

10. If $G(x)=h(p(x))$ $\quad h(5)=1 \quad h'(5)=2$

$h'(1)=3 \quad p(1)=5$

$p'(1)=7$

$G'(1)=h'(p(x))\cdot p'(x)$

$=h'(5)\cdot p'(1)$

$=2\cdot 7$

$=\boxed{14}$