2.7 pg. 107-108

1, 2ac, 3ac, 5a, 6

9 Challenge

#1. (a) $x^2 - y^2 = 1$

$\frac{dy}{dx}$: $2x - 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{x}{y}$$

(b) $x^3 + y^3 = 6$

$$3x^2 + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x^2}{y^2}$$

(c) $xy = 4$

$$x\frac{dy}{dx} + (1)y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

(d) $x^2 + xy + y^2 = 1$

$$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$-(2x + y) = \frac{dy}{dx}(x + 2y)$$

$$\frac{dy}{dx} = \frac{-2x - y}{x + 2y} = -\frac{y + 2x}{x + 2y}$$

$\cancel{dx}$

$2x \cancel{-\frac{y}{x}} \cancel{+2y\frac{dy}{dx}} = 0$

e) $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2\frac{dy}{dx} = 6\left[x\frac{dy}{dx} + y\right].$$

$$3x^2 - 6y = 6x\frac{dy}{dx} - 3y^2\frac{dy}{dx}$$

$$3x^2 - 6y = (6x - 3y^2)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 6y}{6x - 3y^2} = \frac{\cancel{3}(x^2 - 2y)}{\cancel{3}(2x - y^2)}$$

f) $2xy^2 - y^3 = x^2$

$$2\left[x \cdot 2y\frac{dy}{dx} + (1)y^2\right] - 3y^2\frac{dy}{dx} = 2x.$$

$$4xy\frac{dy}{dx} + 2y^2 - 3y^2\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}$$

(g) $x^{1/2} + y^{1/2} = 1$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}} \cdot \frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{2\sqrt{y}}{2\sqrt{x}} = -\frac{\sqrt{y}}{\sqrt{x}}$$

(h) $\frac{2x}{x+y} = y$

$$\frac{(x+y)(2) - (2x)\left(1 + \frac{dy}{dx}\right)}{(x+y)^2} = \frac{dy}{dx}$$

$$\cancel{2x} + 2y \cancel{- 2x} - 2x\frac{dy}{dx} = (x+y)^2\frac{dy}{dx}$$

$$2y = \left[(x+y)^2 + 2x\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2y}{\left[(x+y)^2 + 2x\right]}$$

2. a) $x^2 + 4y^2 = 5$ at $(1, -1)$

$$2x + 8y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{8y} = -\frac{x}{4y}.$$

at $(1,-1)$ $m = -\frac{(1)}{4(-1)} = \frac{1}{4}$

c) $x^2 + x^3y^2 - y^3 = 13$ $(1, -2)$

$$2x + x^3 \cdot 2y\left(\frac{dy}{dx}\right) + 3x^2 \cdot y^2 - 3y^2\frac{dy}{dx} = 0$$

$$2x + 3x^2y^2 = -2x^3y \cdot \frac{dy}{dx} + 3y^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x + 3x^2y^2}{3y^2 - 2x^3y}$$

at $(1,-2)$ $m = \frac{2(1) + 3(1)^2(-2)^2}{3(-2)^2 - 2(1)^3(-2)}$

$$= \frac{2 + 12}{12 + 4} = \frac{+14}{16} = \cancel{\frac{4}{4}}\ \frac{7}{8}$$

2.7

3(a) $2x^2 - y^2 = 1 \quad (-1,-1)$

$4x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2x}{y}$ at $(-1)(-1)$ $= \frac{-2}{-1} = \underline{\underline{2}}$.

$y + 1 = 2(x+1)$

$\boxed{2x - y + 1 = 0}$

(c) $y^5 + x^2y^3 = 10 \quad (-3,1)$

$5y^4\frac{dy}{dx} + x^2 \cdot 3y^2\frac{dy}{dx} + 2xy^3 = 0$

$\frac{dy}{dx} = -\frac{\cancel{5y^4} + 2xy^3}{3x^2y^2 + 5y^4}$ at $(-3,1)$

$= -\frac{\cancel{5(1)}^4 + 2(-3)(1)^3}{3(-3)^2(1)^2 + 5(1)^4} = \frac{+6}{32} = \frac{+3}{16}$

$= \cancel{-\frac{5-6}{27}} = \cancel{\frac{1}{27}}$

$y - 1 = \frac{+3}{16}(x+3)$ $\quad 16y - 16 = +3x + 9$

$\cancel{27y - 27 = x + 3}$ $\quad \boxed{3x - 16y + 25 = 0}$

$\cancel{x - 27y + 30 = 0}$ $\quad$ <u>aargh</u>.

5(a) $x^2+y^2+2x-4y-20=0 \quad (2,-2)$

$$2x+2y\frac{dy}{dx}+2-4\frac{dy}{dx}+0=0$$

$$2x+2=4\frac{dy}{dx}-2y\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{2x+2}{4-2y} \qquad (2,-2)$$

$$=\frac{2(2)+2}{4-2(-2)}=\frac{6}{8}=\frac{3}{4}$$

$$y+2=\frac{3}{4}(x-2)$$

$$4y+8=3x-6$$

$$\boxed{3x-4y-14=0}$$

6. $2(x^2+y^2)^2 = 25(x^2-y^2)$

a) $4(x^2+y^2)(2x+2y\frac{dy}{dx}) = 25(2x-2y\frac{dy}{dx})$

$4(2x^3+2x^2y\frac{dy}{dx}+2xy^2+2y^3\frac{dy}{dx}) = 50x-50y\frac{dy}{dx}$

$8x^3+8x^2y\frac{dy}{dx}+8xy^2+8y^3\frac{dy}{dx} = 50x-50y\frac{dy}{dx}$

$8x^3+8x^2y^2-50x = -8x^2yy'-8y^3y'-50yy'$

$\frac{8x^3+8x^2y^2-50x}{-8x^2y-8y^3-50y} = y'$

$y' = \frac{\cancel{2}x(4x^2+4\cancel{x}y^2-25)}{-\cancel{2}y(\cancel{4}x^2+4y^2+25)}$

$= -\frac{x[4(x^2+y^2)-25]}{y[4(x^2+y^2)+25]}$

(b) $(-3,1)$ $y' = -\frac{(-3)(4(9+1)-25)}{(4(9+1)+25)}$

$= \frac{3(15)}{65} = \frac{9}{13}$

$y-1 = \frac{9}{13}(x+3)$

$13y-13 = 9x+27$

$\boxed{9x-13y+40=0}$

☆ c) want $m=0$: can only happen when

$x[4(x^2+y^2)-25]=0$

$x=0$ or $x^2+y^2=\frac{25}{4}$

but $x \neq 0$ or
div. by zero error

Challenge

9. $x[f(x)]^3 + x^2 f(x) = 3$

$$x \cdot 3[f(x)]^2 \cdot f'(x) + 1[f(x)]^3 + x^2 \cdot f'(x) + 2x \cdot f(x) = 0$$

$$f'(2) = 2 \cdot 3[f(2)]^2 \cdot f'(2) + [f(2)]^3 + 2^2 f'(2) + 2(2) f(2) = 0$$

$$= 2 \cdot 3[1]^2 f'(2) + [1]^3 + 4 f'(2) + 4(1) = 0$$

$$6f'(2) + 1 + 4f'(2) + 4 = 0$$

$$10f'(2) = -5$$

$$f'(2) = \frac{-5}{10}$$

$$f'(2) = \frac{-1}{2}$$