#1 all , 2ab, 3, 4, 7a, 9ab

Exercise 2.8 pg 111-112

#1. a) $f'(x) = 5x^4 - 8x$
$f''(x) = 20x^3 - 8$

b) $g'(x) = 28x^3 + 36x^2 - 4$
$g''(x) = 84x^2 + 72x$

c) $f(t) = 2t - \frac{1}{t+1}$

$f'(t) = 2 + (t+1)^{-2} = 2 + \frac{1}{(t+1)^2}$

$f''(t) = \frac{-2}{(t+1)^3}$

d) $g(t) = 4t^{-1/2}$

$g'(t) = \frac{-2}{t^{3/2}}$ $\quad g''(t) = \frac{+3}{t^{5/2}}$

e) $y' = 8(2x+1)^7 \cdot 2$
$= 16(2x+1)^7$

$y'' = 112(2x+1)^6 \cdot 2$
$= 224(2x+1)^6$

f) $y = t^3 + t^{-3}$
$y' = 3t^2 - 3t^{-4}$
$y'' = 6t + 12t^{-5}$

g) $y = (x^2+1)^{1/2}$
$y' = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$= \frac{x}{\sqrt{x^2+1}}$

$y'' = \frac{(x^2+1)^{1/2} \cdot (1) - \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x \cdot x}{(\sqrt{x^2+1})^2}$

$= \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{1}{(x^2+1)^{3/2}}$

h) $y' = \dfrac{(t-1)(1) - t(1)}{(t-1)^2}$

$= \dfrac{-1}{(t-1)^2}$

$y'' = +2(t-1)^{-3} = \dfrac{+2}{(t-1)^3}$

2. a) $f'(x) = -12 + 8x - 3x^2$

$f''(x) = 8 - 6x$

$f'''(x) = \underline{\underline{-6}}$

c) $y' = -6(4-x)^{-3}(-1)$

$= \dfrac{6}{(4-x)^3} = 6(4-x)^{-3}$

$y'' = -18(4-x)^{-4}(-1)$

$= 18(4-x)^{-4}$

$y''' = 72(4-x)^{-5}$

3. $y' = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

$y'' = 20x^3 + 12x^2 + 6x + 2$

$y''' = 60x^2 + 24x + 6$

$y^{(4)} = 120x + 24$

$y^{(5)} = 120$

$y^{(6)} = 0$

4. $f(x) = (1+x^3)^{1/2}$ 2.8

$$f'(x) = \tfrac{1}{2}(1+x^3)^{-1/2}(3x^2)$$

$$= \frac{3x^2}{2(1+x^3)^{1/2}}$$

$$f''(x) = \frac{2(1+x^3)^{1/2}(6x) - 2(\tfrac{1}{2})(1+x^3)^{-1/2}(3x^2)(3x^2)}{4(1+x^3)}$$

$$= \frac{12x(1+x^3)^{1/2} - 9x^4(1+x^3)^{-1/2}}{4(1+x^3)}$$

$$f''(2) = \frac{12(2)(1+2^3)^{1/2} - 9(2)^4(1+2^3)^{-1/2}}{4(1+2^3)}$$

$$= \frac{24 \cdot 3 - 9 \cdot 16 \cdot 1/3}{4 \cdot 9}$$

$$= \frac{72-48}{36} = \frac{24}{36} = \boxed{\frac{2}{3}}$$

7a) $x^4 + y^4 = 1$

$$\frac{d}{dx}\left(x^4 + y^4\right) = \frac{d}{dx}(1)$$

$$4x^3 + 4y^3 y' = 0$$

$$y' = \frac{-4x^3}{4y^3} \qquad y' = -\frac{x^3}{y^3}$$

$$y'': \frac{d}{dx} y' = \frac{d}{dx}\left(-\frac{x^3}{y^3}\right)$$

$$y'' = -\frac{y^3 \cdot 3x^2 - x^3 \cdot 3y^2 y'}{y^6}$$

$$= -\frac{3x^2y^3 - 3x^3y^2(-x^3/y^3)}{y^6}$$

$$= -\frac{3x^2y^3 + \frac{3x^6}{y}}{y^6}$$

$$= -\frac{3x^2y^4 + 3x^6}{y^7}$$

$$= -\frac{3x^2(y^4 + x^4)}{y^7} \quad \text{but } x^4 + y^4 = 1$$

so $y'' = -\dfrac{3x^2}{y^7}$

9a) $f(x) = g(x)h(x)$

$$f' = g \cdot h' + g'h$$

$$f'' = gh'' + g'h' + g'h' + g''h$$

$$f'' = gh'' + 2g'h' + g''h$$

b) $f''' = gh''' + g'h'' + 2g'h'' + 2g''h' + g''h' + g'''h$

$$f''' = gh''' + 3g'h'' + 3g''h' + g'''h$$