

RATIONAL EXPRESSIONS

- A **RATIONAL** expression is an algebraic fraction

Ex: $\frac{2x}{5y}$ or $\frac{3x^2+4x+1}{x^2-1}$ or $\frac{5x+1}{2x}$

- Since we are not allowed to divide by zero, the denominator of a rational expression cannot equal zero. This means that the variable in the denominator cannot take on a value that would make the denominator 0

Ex: $\frac{2x}{2y}$ → y cannot ever equal zero since $2 \cdot (0) = 0$ which would give us $\frac{2x}{0}$ which is not allowed

So, for the expression $\frac{2x}{2y}$ we state

the restriction or non-permissible value (NPV) to be $y \neq 0$

Ex $\frac{3x}{x-5}$

also called the restriction
NPV: $x \neq 5$

$\frac{3x}{2x-3}$

NPV: $x \neq \frac{3}{2}$
from:
 $2x-3 \neq 0$
 $+3 \quad +3$
 $\frac{2x+3}{2} \neq \frac{3}{2}$
 $x \neq \frac{3}{2}$

$\frac{2xy}{3x+y}$

NPV: $x \neq -\frac{1}{3}y$
from: $3x+y \neq 0$
 $\frac{3x}{3} \neq \frac{-y}{3}$
 $x \neq -\frac{1}{3}y$

$\frac{3}{x^2-4}$

→ Rewrite as:

$\frac{3}{(x-2)(x+2)}$

NPV: $x \neq 2$ or -2

(usually written as $x \neq \pm 2$)

since $x=2$ or -2 would make the denominator zero

Ex 5

$$\frac{16x^2 - 9y^2}{8x - 6y} = \frac{(4x-3y)(4x+3y)}{2(4x-3y)}$$

factor

$$= \frac{(4x+3y)}{2}$$

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$$= \frac{(4x+3y)(4x+3y)}{2(4x-3y)}$$

NPV:
 $4x \neq 3y$
 so
 $x \neq \frac{3}{4}y$

$$= \frac{(4x+3y)}{2}$$

Ex 6

$$\frac{6-2m}{m^2-9} = \frac{2(3-m)}{(m-3)(m+3)}$$

NPV:
 $m \neq \pm 3$

almost but NOT quite the same so we try factoring a -1 out of the top

$$= \frac{2(-1(-3+m))}{(m-3)(m+3)} = \frac{2(-1)(m-3)}{(m-3)(m+3)}$$

$$= \frac{-2(m-3)}{(m-3)(m+3)}$$

$$= \frac{-2}{m+3}$$

Examples from Text assignment

3a) $\frac{-4}{x}$ $x=0$ would make the denominator zero

4a) $\frac{3a}{4-a}$ NPV: $a \neq 4$

since that would make the denominator zero

6a) $\frac{2\cancel{c}(c-5)}{3\cancel{c}(c-5)}$ NPV: $c \neq 0$ or 5

$$= \frac{2 \cdot 1 \cdot 1}{3 \cdot 1 \cdot 1} = \left(\frac{2}{3}\right)$$

8c) $\frac{b^2 + 2b - 24}{2b^2 - 72} = \frac{(b+6)(b-4)}{2(b^2 - 36)}$

$$= \frac{\cancel{(b+6)}(b-4)}{2(b-6)\cancel{(b+6)}}$$

NPV: $b \neq \pm 6$

$$= \frac{(b-4)}{2(b-6)}$$