Composite functions

- Composite functions occur when we substitute one function into another
- For example:

$f(x) = 3x-7$ and $g(x) = 2x^2+3$

$f(3) = 3(3) - 7$
$= 2$

$f(g(x)) = 3(2x^2+3) - 7$
$= 6x^2+9-7 = 6x^2+2$

means wherever there is an 'x' in f(x), we substitute in g(x)

Ex 2 $f(x) = 3x-7$ $g(x) = 2x^2+3$

$g(f(x)) = 2(3x-7)^2 + 3$
$= 2(3x-7)(3x-7) + 3$
$= 2(9x^2 - 21x - 21x + 49) + 3$
$= 2(9x^2 - 42x + 49) + 3$
$= 18x^2 - 84x + 98 + 3$

$g(f(x)) = 18x^2 - 84x + 101$

Terminology / Symbols

$f(g(x)) = (f \circ g)(x)$

"f" "dot" "g" of x

$f(x) \cdot g(x) = (f \cdot g)(x)$

or $(fg)(x)$

$(g \circ f)(x)$ means $g(f(x))$

$(g \circ f)(3) \Rightarrow g(f(3))$

$g(x) = 3x^2 - 2$ $\quad f(x) = 2x - 1$

$g(5) = 3(5)^2 - 2$ $\quad f(3) = 2(3) - 1 = 5$

$g(f(3)) = g(5) = 73$

or Find $g(f(x))$

$g(f(x)) = 3(2x-1)^2 - 2$

$g(f(3)) = 3(2(3)-1)^2 - 2$

$= 3(5)^2 - 2$

$= 73$

$f(x)$: $[(7,2)\ (9,12)\ (10,15)\ (11,6)]$

$g(x)$: $[(3,7)\ (5,9)\ (8,12)\ (10,11)]$

$g(f(15)) = 11$

$f(g(3)) = f(7) = 2$

$f(g(10)) = f(11) = 6$

$f(g(8)) = f(12)$ DNE

does not exist

Questions

① $f(x)=4x$ $g(x)=x+6$ $h(x)=x^2$

find:

ANSWERS

(a) $f(g(3))$ 36

(b) $g(h(-2))$ 10

(c) $h(h(2))$ 16

$f(x)=\sqrt{x-1}$ $g(x)=x^2$

STATE RESTRICTIONS

Find $f(g(x)) = \sqrt{x^2-1}$ $x^2-1 \geq 0$, $x^2 \geq 1$

$g(f(x)) = x-1$ $x > 1$

$g(g(x)) = x^4$ None

If $f(x) = 3x+4$ and $g(x) = x^2-1$

FIND:

ANSWERS

① $(f \circ g)(-3)$ $= 28$

② $(f \circ g)(a)$ $= 3a^2+1$

③ $(g \circ f)(a)$ $= 9a^2+24a+15$

④ $(f \circ g)(x)$ $= 3x^2+1$

⑤ $(g \circ f)(x)$ $= 9x^2+24x+15$

⑥ $(g \circ g)(x)$ $= x^4-2x^2$

10.3 Try to get #1-5 done for tomorrow (or farther if you have time)

Ch10 Test
TEST FRIDAY