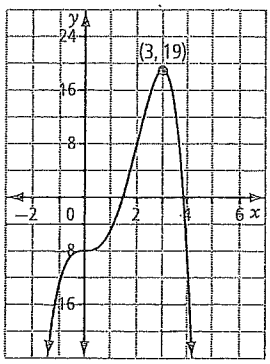
3.1 Characteristics of Polynomial Functions, pages 66–77

1. Complete the chart for each polynomial function.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$	4	Quar	-2	-7
b) $y = 3x^5 + 2x^4 - x^3 + 3$	5	Quin	3	3
c) $g(x) = 0.5x^3 - 8x^2$	3	cubic	.5	0
d) $p(x) = 10$	0	Constant	0	10

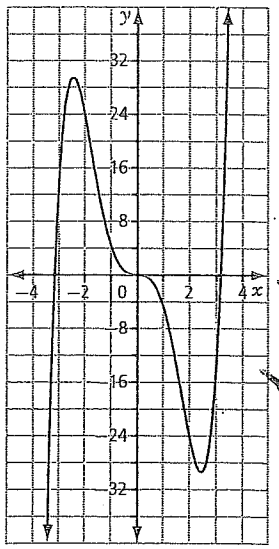
2. For each of the following,
   - determine whether the graph represents on odd-degree or an even-degree polynomial function
   - determine whether the leading coefficient of the corresponding function is positive or negative
   - state the number of x-intercepts
   - state the domain and range

a)



even degree
2 x-intercepts
D: R
R: $y \leq 19$
negative leading coef

b)



odd degree
positive leading coef
3 intercepts
D: R
R: R

3. The distance, d, in metres, travelled by a boat from the moment it leaves shore can be modelled by the function $d(t) = 0.002t^3 + 0.05t^2 + 0.3t$, where t is the time, in seconds.

a) What is the degree of the function $d(t)$?

3

b) What are the leading coefficient and constant of this function? What does the constant represent?

.002 0 distance boat from shore at time 0

c) Describe the end behaviour of the graph of this function.

extends from Quad 3 → 1

d) What are the restrictions on the domain of this function? Explain why you selected those restrictions.

$t \geq 0$ can't have negative time

e) What distance has the boat travelled after 15 s?

$t = 15$ $d(15) = 22.5$ m

f) Make a sketch of what you think the function will look like. Then, graph the function using technology. How does it compare to your sketch?

3.2 The Remainder Theorem, pages 78–83

4. a) Use long division to divide $5x^3 - 7x^2 - x + 6$ by $x - 1$.

Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

$5x^2 - 2x - 3 + \frac{3}{x-1}$

b) Identify any restrictions on the variable.

$x \neq 1$

c) Write the corresponding statement that can be used to check the division. Then, verify your answer.

$(x-1)(5x^2-2x-3)+3=$

5. Determine the remainder resulting from each division.

a) $(x^3 + 2x^2 - 3x + 9) \div (x + 3)$

$R = 9$

b) $(2x^3 + 7x^2 - x + 1) \div (x + 2)$

$R = 15$

c) $(x^3 + 2x^2 - 3x + 5) \div (x - 3)$

$R = 41$

d) $(2x^4 + 7x^2 - 8x + 3) \div (x - 4)$

$R = 595$

6. a) Determine the value of m such that when $f(x) = x^4 - mx^3 + 7x - 6$ is divided by $x - 2$, the remainder is -8.

$-8 = (2)^4 - m(2)^3 + 7(2) - 6$

$m = 4$

b) Use the value of m from part a) to determine the remainder when $f(x)$ is divided by $x + 2$.

28

7. When a polynomial $P(x)$ is divided by $x - 2$, the quotient is $x^2 + 4x - 7$ and the remainder is -4. What is the polynomial?

$x^3 + 2x^2 - 15x + 10$

3.3 The Factor Theorem, pages 84–90

8. What is the corresponding binomial factor of a polynomial, $P(x)$, given the value of the zero?

a) $P(7) = 0$

$x - 7$

b) $P(-6) = 0$

$x + 6$

c) $P(c) = 0$

$x - c$

9. Determine whether $x + 2$ is a factor of each polynomial.

a) $x^3 + 2x^2 - x - 2$

Yes

b) $x^4 + 2x^3 - 4x^2 + x + 10$

No

10. What are the possible integral zeros of each polynomial?

a) $x^3 - 5x^2 + 3x - 27$

$\pm 1 \pm 3 \pm 9 \pm 27$

b) $x^3 + 6x^2 + 2x + 36$

$\pm 1 \pm 2 \pm 3 \pm 4 \pm 6 \pm 9 \pm 12 \pm 18 \pm 36$

11. Factor fully.

a) $x^3 - 4x^2 + x + 6$

$(x-3)(x-2)(x+1)$

$x = -1, 2, 3$

b) $3x^3 - 5x^2 - 26x - 8$

$(x-4)(x+2)(3x+1)$

$x = 4, -2, -\frac{1}{3}$

c) $5x^4 + 12x^3 - 101x^2 + 48x + 36$

$(x-3)(x-1)(x+6)(5x+2)$

$x = 3, 1, -6, -\frac{2}{5}$

d) $2x^4 + 5x^3 - 8x^2 - 20x$

$x(x-2)(x+2)(2x+5)$

$x = 0, 2, -2, -\frac{5}{2}$

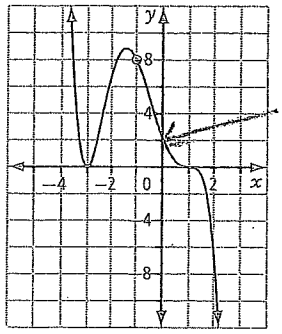
12. Rectangular blocks of ice are cut up and used to build the front entrance of an ice castle. The volume, in cubic feet, of each block is represented by $V(x) = 5x^3 + 7x^2 - 8x - 4$, where x is a positive real number. What are the factors that represent possible dimensions, in terms of x, of the blocks?

$x-1, x+2, 5x+2$

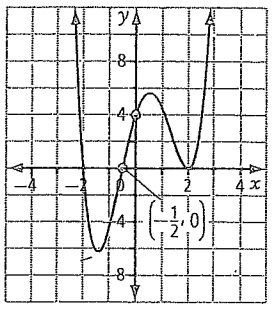
3.4 Equations and Graphs of Polynomial Functions, pages 91–102

13. For each graph of a polynomial function, determine
- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and their multiplicity
- the intervals where the function is positive and the intervals where it is negative
- the equation for the polynomial function

a)



b)



14. a) Given the function $y = x^5$, list the parameters of the transformed polynomial function $y = -2\left(\frac{1}{3}(x-1)\right)^5 + 4$ and describe how each parameter transforms the graph of the function $y = x^5$.

b) Determine the domain and range for the transformed function.

15. Determine the equation with least degree for a cubic function with zeros −2 (multiplicity 2) and 3 (multiplicity 1), and y-intercept 36.