2.6 The Chain Rule

If the derivatives of $g(x)$ and $f(g(x))$ both exist and if $F = f \circ g$ is the composite fn defined by $F(x) = f(g(x))$ then $F'(x)$ exists and is given by:

$\Rightarrow F'(x) = f'(g(x)) \cdot g'(x)$

or

$\Rightarrow \star \ \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Lets look at some examples:

Ex 1: $y = (2x+3)^2$

Think: $y = x^2$
$y' = 2x$

Does that work when the fn is more complicated?

$y = (2x+3)^2$

$y' \overset{?}{=} 2(2x+3) = \boxed{4x+6}$

check it by expanding

$y = (2x+3)^2 = 4x^2 + 12x + 9$

$y' = 8x + 12$

They don't match but the second one is 2x the first.

→ The derivative of the fn inside the bracket is 2, so if we multiply the answer by the derivative it works out.

Ex $y = (2x+3)^2$

$y = f(g(x))$ where $f(x) = x^2$, $g(x) = 2x+3$

The chain rule

$y' = f'(g(x)) \cdot g'(x)$

$= 2(2x+3) \cdot 2$

$= \underline{\underline{8x+12}}$

$f'(x) = 2x$

$f'(g(x)) = 2(2x+3)$

$g'(x) = 2$

Going back to

$y = (x)^2$

$y = 2(x) \cdot (1) = 2x$

Ex 2 $\quad h(x) = (2x^2+3)^{1/2}$

$h(x) = f(g(x))$ then

$f(x) = x^{1/2}$

$g(x) = 2x^2+3$

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(g(x)) = \frac{1}{2}(2x^2+3)^{-1/2}$

$g'(x) = 4x$

$$h'(x) = \underbrace{f'(g(x))}\cdot g'(x)$$

$$h'(x) = \frac{1}{2}(2x^2+3)^{-1/2} \cdot 4x$$

a then simplify!!

$$h'(x) = \frac{\cancel{4}^{2}x}{\cancel{2}_{1}\sqrt{2x^2+3}} = \frac{2x}{\sqrt{2x^2+3}}$$

Ex 3. $y = (x^2 - x + 2)^8$

$y' = 8(x^2 - x + 2)^7 (2x - 1)$

Ex 4 $y = \frac{3}{2}(2x^2 + 3x + 5)^{4/3}$

$y' = 2(2x^2 + 3x + 5)^{1/3} \cdot (4x + 3)$

Ex 5 $y = \left(\frac{2t-1}{t+2}\right)^6$

Method I

$y' = 6\left(\frac{2t-1}{t+2}\right)^5 \cdot \left[\frac{(t+2)(2) - (2t-1)(1)}{(t+2)^2}\right]$

is a quotient so we need the quotient rule!

$= 6\left(\frac{2t-1}{t+2}\right)^5 \cdot \left[\frac{\cancel{2t} + 4 - \cancel{2t} + 1}{(t+2)^2}\right]$

$= 6\left(\frac{2t-1}{t+2}\right)^5 \cdot \left[\frac{5}{(t+2)^2}\right]$

$\left(\frac{2t-1}{t+2}\right)^5 = \frac{(2t-1)^5}{(t+2)^5}$

$$= \frac{30(2t-1)^5}{(t+2)^7}$$

<u>Method II</u> $y = \left(\frac{2t-1}{t+2}\right)^6$

Let's let $u = \frac{2t-1}{t+2}$

then $\underline{\underline{u'}} = \frac{\overbrace{(t+2)(2)}^{2t+4} - \overbrace{(2t-1)(1)}^{-2t+1}}{(t+2)^2}$

$= \frac{5}{(t+2)^2}$

Then $y = \underline{\underline{u}}^6$

$y' = 6(u^5) \cdot (u')$

$= 6\left(\frac{2t-1}{t+2}\right)^5 \cdot \left(\frac{5}{(t+2)^2}\right)$

$= \frac{30(2t-1)^5}{(t+2)^7}$

Then substitute the $f^{\underline{n}}$ back in for $\underline{u}$

Find $\left.\frac{dy}{dx}\right]_{x=1}$ if $\boxed{y = u^{10} + u^5 + 2}$

and

$u = 1 - 3x^2$

$\frac{du}{dx}$ or $u' = -6x$

$u'(1) = -6$

$$\frac{dy}{dx} = 10u^9 \cdot u' + 5u^4 \cdot u'$$

or

$$= 10u^9 \frac{du}{dx} + 5u^4 \frac{du}{dx}$$

these are the same!

$$\frac{dy}{dx} = 10(1-3x^2)^9(-6x) + 5(1-3x^2)^4(-6x)$$

$$= 5(-6x)(1-3x^2)^4\left[2(1-3x^2)^5 + 1\right]$$

$$= -30x\,(1-3x^2)^4\left[2(1-3x^2)^5 + 1\right]$$

$$\left.\frac{dy}{dx}\right]_{x=1} = -30(1)(1-3(1)^2)^4\left[2(1-3(1)^2)^5 + 1\right]$$

$$= -30(16)(-64+1)$$

$$= 30240$$

27 WED : 2.6 ü

28 Thurs : 2.7

1 FRI : Work Block / W/S's

4 MON : 2.8

5 TUES : Work / Review

6 WED : Work / Review

7 THURS : TEST

8 Fri : Free day Stay home.

Now You Should Get to Work!!

Calculus takes it to the Limit