Ch 2 Review Questions (w/s)

#1. $y' = -2x$

$y'' = -2$

#2 $y' = x^2 - 1$

$y'' = 2x$

#3. $y' = 2$

$y'' = 0$

#4 $y = x^2 + x + 1$

$y' = 2x + 1$

$y'' = 2$

#5 $y' = x^2 + x + 1$

$y'' = 2x + 1$

#6. $y' = -1 + 2x - 3x^2$

$y'' = 2 - 6x$

#7 $y' = 4x^3 - 21x^2 + 4x$

$y'' = 12x^2 - 42x + 4$

#8. $y' = 15x^2 - 15x^4$

$y'' = 30x - 60x^3$

#9. $y' = -8x^{-3} - 8$

$y'' = 24x^{-4}$

#10. $y' = -x^{-5} + x^{-4} - x^{-3} + x^{-2}$

$y'' = 5x^{-6} - 4x^{-5} + 3x^{-4} - 2x^{-3}$

#13 $y = \dfrac{2x+5}{3x-2}$

$$y' = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$$

$$= \frac{6x-4-6x-15}{(3x-2)^2}$$

$$= \boxed{\frac{-19}{(3x-2)^2}}$$

#14 $y = \dfrac{x^2+5x-1}{x^2}$

$$y' = \frac{x^2\cdot(2x+5) - 2x(x^2+5x-1)}{(x^2)^2}$$

$$= \frac{2x^3+5x^2-2x^3-10x^2+2x}{x^4}$$

$$= \frac{-5x^2+2x}{x^4}$$

$$= \frac{x(2-5x)}{x^4} = \boxed{\frac{2-5x}{x^3}}$$

#15 $y = \dfrac{(x-1)(x^2+x+1)}{x^3}$

$$y' = \frac{x^3 \cdot [(x-1)\cdot(2x+1) + (1)(x^2+x+1)] - 3x^2(x-1)(x^2+x+1)}{(x^3)^2}$$

$$y' = \frac{x^3[2x^2-x-1+x^2+x+1] - 3x^2[x^3+x^2+x-x^2-x-1]}{x^6}$$

$$= \frac{x^2[x[3x^2] - 3[x^3-1]]}{x^6}$$

$$= \frac{3x^3-3x^3+3}{x^4}$$

$$= \boxed{\frac{3}{x^4}}$$

#16 $y = (1-x)(1+x^2)^{-1}$

$$y' = (1-x)(-1(1+x^2)^{-2} \cdot (2x)) + (-1)(1+x^2)^{-1}$$

$$= \frac{(1-x)(-2x)}{(1+x^2)^2} - \frac{1}{(1+x^2)} \cdot \frac{(1+x^2)}{(1+x^2)}$$

$$= \frac{-2x+2x^2-1-x^2}{(1+x^2)^2}$$

$$= \frac{x^2-2x-1}{(1+x^2)^2}$$

#17 $y' = \dfrac{(1-x^3)(2x) - (-3x^2)(x^2)}{(1-x^3)^2}$

$$y' = \frac{2x - 2x^4 + 3x^4}{(1-x^3)^2}$$

$$y' = \frac{2x + x^4}{(1-x^3)^2} \quad \text{OR} \quad \frac{x(2+x^3)}{(1-x^3)^2}$$

#18. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

$$y' = \frac{(x^{1/2}+1)(\frac{1}{2}x^{-1/2}) - (\frac{1}{2}x^{-1/2})(x^{1/2}-1)}{(\sqrt{x}+1)^2}$$

$$y' = \frac{\dfrac{x^{1/2}+1}{2x^{1/2}} - \dfrac{x^{1/2}-1}{2x^{1/2}}}{(\sqrt{x}+1)^2}$$

$$y' = \frac{x^{1/2}+1 - x^{1/2}+1}{2\sqrt{x}(\sqrt{x}+1)^2}$$

$$y' = \frac{\cancel{2}}{\cancel{2}\sqrt{x}(\sqrt{x}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$

19. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

METHOD I: [expand first]

$$y = \frac{x^2+3x+2}{x^2-3x+2}$$

$$y' = \frac{(x^2-3x+2)(2x+3) - (x^2+3x+2)(2x-3)}{(x^2-3x+2)^2}$$

$$y' = \frac{2x^3+3x^2-6x^2-9x+4x+6-(2x^3-3x^2+6x^2-9x+4x-6)}{(x^2-3x+2)^2}$$

$$y' = \frac{2x^3-3x^2-5x+6-2x^3-3x^2+5x+6}{(x^2-3x+2)^2}$$

$$y' = \frac{-6x^2+12}{(x^2-3x+2)^2}$$

METHOD II: [Product Rule within the quotient rule]

$$y' = \frac{(x-1)(x-2)[1(x+2)+1(x+1)] - (x+1)(x+2)[1(x-2)+1(x-1)]}{[(x-1)(x-2)]^2}$$

$$= \frac{(x^2-3x+2)(2x+3) - (x^2+3x+2)(2x-3)}{[x^2-3x+2]^2}$$

⋮

and the rest is the same...

#23 $u(0)=5,\ u'(0)=-3,\ v(0)=-1,\ v'(0)=2$

a) $\frac{d}{dx}(uv) = u \cdot v' + u' \cdot v$

$= 5(2) + (-3)(-1)$

$= 10 + 3 = \underline{\underline{13}}$

b) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - v' \cdot u}{v^2}$

$= \frac{(-1)(-3) - (2)(5)}{(-1)^2}$

$= 3 - 10 = \underline{\underline{-7}}$

c) $\frac{d}{dx}\left(\frac{v}{u}\right) = \frac{u \cdot v' - u' \cdot v}{u^2}$

$= \frac{5(2) - (-3)(-1)}{(5)^2} = \frac{10-3}{25} = \underline{\underline{\frac{7}{25}}}$

d) $\frac{d}{dx}(7v - 2u) = 7v' - 2u'$

$= 7(2) - 2(-3)$

$= 14 + 6 = \underline{\underline{20}}.$

#25. $y = x^2 + 5x$ at $x = 3$

$y' = 2x + 5$

$y'(3) = 6 + 5 = \underline{11}$ so (iii)

#27. $y' = 3x^2 - 3$

$y'(2) = 3(2)^2 - 3$
$= 9$

so $m = \frac{-1}{9}$ $\therefore$ $y - 3 = -\frac{1}{9}(x - 2)$

$9y - 27 = -x + 2$

$x + 9y - 29 = 0$

#32 $y = \frac{8}{4 + x^2} = 8(4 + x^2)^{-1}$

$y' = -8(4 + x^2)^{-2}(2x) = \frac{-16x}{(4 + x^2)^2}$

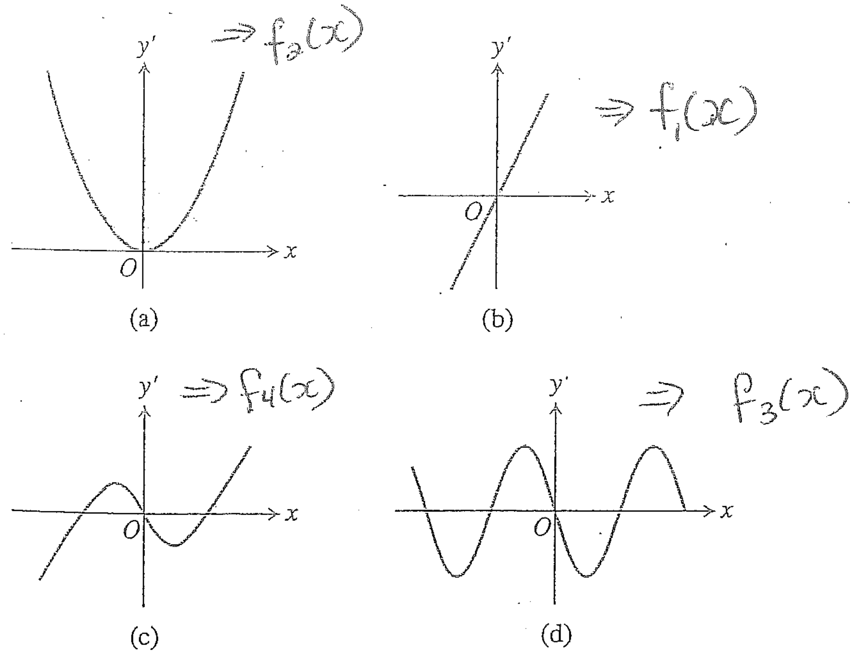
$y'(2) = \frac{-16(2)}{(4 + 2^2)^2} = \frac{-16 \cdot 2}{64} = \frac{-1}{2}$

$y - 1 = -\frac{1}{2}(x - 2)$

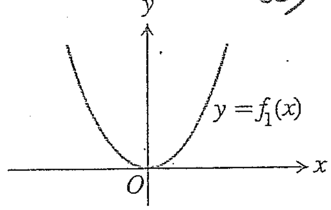
$y - 1 = -\frac{1}{2}x + 1 \Rightarrow y = -\frac{1}{2}x + 2$

OR $x + 2y - 4 = 0$

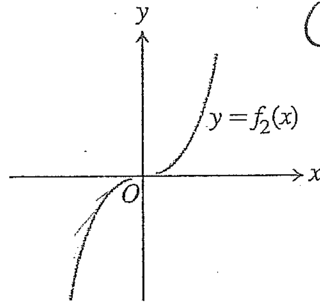
In Exercises 7–10, match the graph of the function with the graph of the derivative shown here:



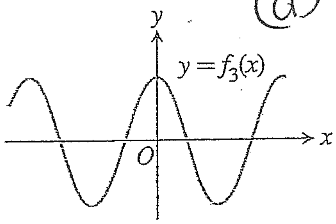
7. (b)



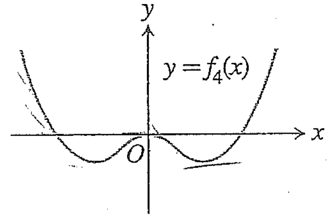
8. (a)



9. (d)



10. (c)



9. $y = (x + \sqrt{x})^{-2}$

$y' = -2(x + \sqrt{x})^{-3}(1 + \frac{1}{2}x^{-1/2})$

$= \dfrac{-2\left(1 + \frac{1}{2\sqrt{x}}\right)}{(x + \sqrt{x})^3}$

$= \dfrac{-2 + \frac{-1}{\sqrt{x}}}{(x + \sqrt{x})^3} = \dfrac{\frac{-2\sqrt{x} - 1}{\sqrt{x}}}{(x + \sqrt{x})^3}$

$= \dfrac{-2\sqrt{x} - 1}{\sqrt{x}(x + \sqrt{x})^3}$

12. $y = x^3(2x-5)^4$

$y' = x^3 \cdot 4(2x-5)^3(2) + 3x^2(2x-5)^4$

$= 8x^3(2x-5)^3 + 3x^2(2x-5)^4$

OR

$= x^2(2x-5)^3(8x + 3(2x-5))$

$= x^2(2x-5)^3(14x-15)$

#15 $y = 3(2x+1)^{-1/2}$

$$y' = -\frac{3}{2}(2x+1)^{-3/2}(2)$$

$$= \frac{-3}{(2x+1)^{3/2}}$$

#16. $y = \frac{x}{(1+x^2)^{1/2}}$

$$y' = \frac{(1+x^2)^{1/2}(1) - (x)\left(\frac{1}{2}(1+x^2)^{-1/2} \cdot (2x)\right)}{(1+x^2)^{1/2}}$$

$$y' = \frac{(1+x^2)^{1/2} - \frac{x^2}{(1+x^2)^{1/2}}}{(1+x^2)}$$

$$y' = \frac{\frac{(1+x^2) - x^2}{(1+x^2)^{1/2}}}{(1+x^2)} = \frac{1}{(1+x^2)^{3/2}}$$

#33 $f(u) = u^5 + 1$ $\quad u = g(x) = \sqrt{x}$ at $x = 1$

$f' = 5u^4 \cdot u'$ $\quad u(1) = \sqrt{1} = 1$

$f'(1) = 5(1)^4 \cdot \left(\frac{1}{2}\right)$ $\quad u' = \frac{1}{2}x^{-1/2}$

$= \frac{5}{2}$ $\quad u'(1) = \frac{1}{2}(1)^{-1/2} = \frac{1}{2}$

OR: $f(x) = (\sqrt{x})^5 + 1$

$$f'(x) = 5(\sqrt{x})^4 \cdot \frac{1}{2}(x)^{-1/2}$$

$$= \frac{5(\sqrt{x})^4}{2\sqrt{x}} = \frac{5}{2}(\sqrt{x})^3$$

$$f'(1) = \frac{5}{2}(\sqrt{1})^3 = \frac{5}{2}$$

#34 $f(u) = 1 - \frac{1}{u}$, $u = \frac{1}{1-x}$ at $x = -1$

$f'(u) = \frac{1}{u^2} \cdot u'$

(from $f = 1 - u^{-1}$

$f' = 0 - (-1)u^{-2} \cdot u'$

$= \frac{u'}{u^2}$

$u' = -(1-x)^{-2}(-1)$

$= \frac{1}{(1-x)^2}$

$u(-1) = \frac{1}{1-(-1)} = \frac{1}{2}$

$f'(-1) = \frac{(1/4)}{(1/2)^2} = 1$

$u'(-1) = \frac{1}{(1-(-1))^2} = \frac{1}{4}$

#37 $f(u) = \dfrac{2u}{u^2+1}$, $u = g(x) = 10x^2 + x + 1$

at $x = 0$

$$u(0) = 10(0)^2 + 0 + 1$$
$$= 1$$

$$u' = 20x + 1$$

$$u'(0) = 20(0) + 1$$
$$= 1$$

$$f' = \frac{(u^2+1)(2 \cdot u') - 2u(2u \cdot u')}{(u^2+1)^2}$$

$$f'(0) = \frac{(1^2+1)(2(1)) - 2(1) \cdot (2(1) \cdot (1))}{(1^2+1)^2}$$

$$= \frac{4 - 4}{4} = 0$$

#38 $f(u) = \left(\frac{u-1}{u+1}\right)^2$ $\quad u = \frac{1}{x^2} - 1$ at $x = -1$

$u(-1) = \frac{1}{(-1)^2} - 1 = \underline{\underline{0}}$

$u' = -2x^{-3}$

$u'(-1) = \frac{-2}{(-1)^3} = \underline{\underline{2}}$

$$f'(u) = 2\left(\frac{u-1}{u+1}\right) \cdot \left[\frac{(u+1)(u') - (u-1)(u')}{(u+1)^2}\right]$$

$$= 2\left(\frac{u-1}{u+1}\right)\left[\frac{u + u' - u + u'}{(u+1)^2}\right]$$

minus

$$= 2\left(\frac{u \cancel{+} 1}{u+1}\right)\left[\frac{2u'}{(u+1)^2}\right]$$

$$f'(-1) = 2\left(\frac{0-1}{0+1}\right)\left[\frac{2(2)}{(0+1)^2}\right]$$

$$= 2(-1)(4) = \underline{\underline{-8}}$$

METHOD 2

#38 $\left(\dfrac{\frac{1}{x^2} - 1 - 1}{\frac{1}{x^2} - 1 + 1}\right)^2$

$$= \left(\frac{\frac{1}{x^2} - 2}{\frac{1}{x^2}}\right)^2$$

$$= \left[\left(\frac{1}{x^2} - 2\right) \cdot \frac{x^2}{1}\right]^2$$

$$= \left(1 - 2x^2\right)^2$$

$$= 2\left(1 - 2x^2\right)\left(-4x\right)$$

$$f(-1) = 2\left(1 - 2(-1)^2\right)(4)$$

$$= 2(1-2)(4)$$

$$2(-1)(4) = -8$$

#56

(a) $y = 2f(x)$

$y' = 2 \cdot f'(x)$.

$y'(2) = 2 \cdot f'(2)$

$= 2(1/3) \quad = 2/3$

(b) $y = f(x) + g(x)$

$y' = f'(x) + g'(x)$

$y'(3) = f'(3) + g'(3)$

$= \cancel{3}\, 2\pi + 5$

$y'(3) = 2\pi + 5$

(c) $y = f(x) \cdot g(x)$

$y' = f(x) \cdot g'(x) + f'(x)\, g(x)$

$y'(3) = f(3) \cdot g'(3) + f'(3)\, g(3)$

$= 3 \cdot 5 + (2\pi)(-4)$

$= \underline{15 - 8\pi}$

d) $y = \dfrac{f(x)}{g(x)}$

$$y' = \frac{g(x) \cdot f'(x) - g'(x)\, f(x)}{[g(x)]^2}$$

$$y'(2) = \frac{(2)(1/3) - (-3)(8)}{(2)^2}$$

$$= \frac{2/3 + 24}{4}$$

$$= \frac{\frac{2}{3} + \frac{72}{3}}{\frac{4}{1}} = \frac{74}{3} \cdot \frac{1}{4} = \frac{37}{6}$$

e) $f(g(x)) = y$

$$y' = f'(g(x))(g'(x))$$

$$y'(2) = f'(g(2)) \cdot g'(2)$$

$$= [f'(2)](-3)$$

$$= \frac{1}{3} \cdot -3 = \underline{-1}$$

f) $\sqrt{f(x)} = y$

$$y' = \frac{1}{2}(f(x))^{-1/2} \cdot f'(x)$$

$$y'(2) = \frac{1}{2}(8)^{-1/2} \cdot (1/3)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{8}} \cdot \frac{1}{3}$$

$$= \frac{1}{6} \cdot \frac{1}{2\sqrt{2}} = \frac{1}{12\sqrt{2}} \text{ or } \frac{\sqrt{2}}{24}$$

g) $y = \dfrac{1}{g^2(x)}$

$$y = [g(x)]^{-2}$$

$$y' = -2[g(x)]^{-3} \cdot g'(x)$$

$$y'(3) = -2[g(3)]^{-3} \cdot g'(3)$$

$$= -2[-4]^{-3} \cdot 5$$

$$= \frac{-10}{-64} = \frac{5}{32}$$

h) $y = (f^2(x) + g^2(x))^{1/2}$ at $x = 2$

$$y' = \frac{1}{2}(f^2(x) + g^2(x))^{-1/2} \cdot (2f(x) \cdot f'(x) + 2g(x) \cdot g'(x))$$

$$= \frac{1}{2}\left(\boxed{f(2)}^2 + \boxed{g(2)}^2\right)^{-1/2} \cdot \left(2\left(f(2) \cdot f'(2) + 2g(2)g'(2)\right)\right)$$

$$= \frac{1}{2}(8^2 + 2^2)^{-1/2}(2 \cdot (8)(1/3) + 2(2)(-3))$$

$$= \frac{1}{2}(68)^{-1/2}\left(\frac{16}{3} - 12\right)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{68}} \cdot \left(\frac{16 - 36}{3}\right)$$

$$= \frac{-20}{6\sqrt{68}} = \frac{-10}{3\sqrt{68}} = \frac{-10}{3 \cdot 2\sqrt{17}}$$

$$= \frac{-5}{3\sqrt{17}}$$