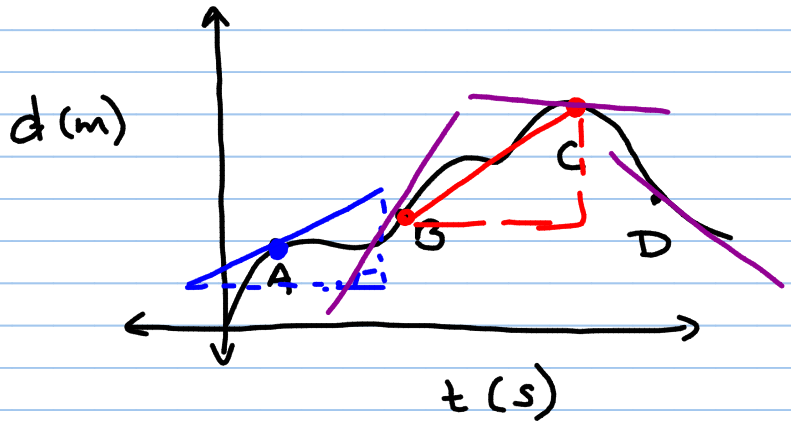
# Chapter 3 ~ Applications of Derivatives

## 3.1 Velocity




NB: The text often uses $s(t)$ for displacement functions. I often use $d(t)$

- Average velocity between B & C?
- Instantaneous velocity? A? B? C?
  - ↳ SLOPE of the tangent line... or the derivative of $d(t)$
- When is the object moving the fastest? Slowest?
  - fastest at B
  - slowest at C

**Ex 1** The position of a particle moving on a straight line is given by:

$$d(t) = 2t^3 - 21t^2 + 60t \qquad (t \geq 0)$$

(a) What is the velocity after 3s? 6s?

1st, find $d'$

$$d'(t) = 6t^2 - 42t + 60$$

$$d'(3) = 6(3)^2 - 42(3) + 60 = -12 \text{ m/s}$$

$$d'(6) = 6(6)^2 - 42(6) + 60 = 24 \text{ m/s}$$

(b) When is the particle at rest?

Want the tangent to have a slope of 0

So: $\frac{1}{6}(6t^2 - 42t + 60 = 0) \cdot \frac{1}{6}$

$$t^2 - 7t + 10 = 0$$

$$(t-5)(t-2) = 0$$

so $t = 2s$ or $5s$

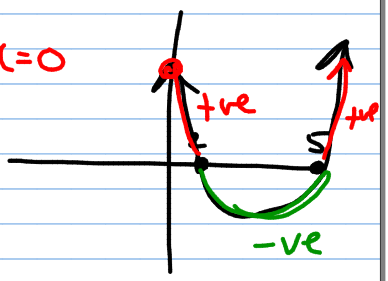
(c) When is the particle moving in the positive direction?

Look at $0 \to 2$ seconds
$2 \to 5$ seconds
$5 \to$ and on

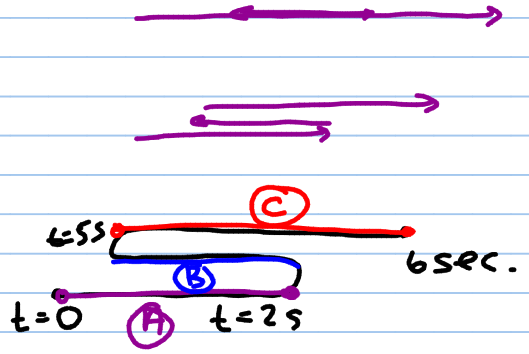
$6(0) - 42(0) + 60 = 60$ when $t = 0$

$6t^2 - 42t + 60 > 0$?

$0 \leq t < 2$ positive
and
$t > 5$ positive



d) What is the total distance traveled in the first 6 seconds?




We have to add up the pieces.

To find the distance, go back to the original distance function:

$$d(t) = 2t^3 - 21t^2 + 60t$$

(A) $0 \to 2s$: $|d(2) - d(0)| = |52 - 0| = 52m$

$d(2) = 2(2)^3 - 21(2)^2 + 60(2)$
$= 16 - 84 + 120 = 52m$

$d(0) = 0$

(B) $2 - 5s$: $|d(5) - d(2)| = |25 - 52| = 27m$

$d(5) = 2(5)^3 - 21(5)^2 + 60(5)$
$= 250 - 525 + 300$
$= 25m$

$d(2) = 52$

(C) $5 - 6s$: $|d(6) - d(5)| = |36 - 25| = 11m$

$d(6) = 2(6)^3 - 21(6)^2 + 60(6)$
$= 432 - 756 + 360 = 36$

Total d: $52 + 27 + 11 = 90m$

## Acceleration:

Is the slope of a line tangent to a velocity-time graph

OR the 2nd derivative of a displacement function.

Ex $s = t^3 + 2t^2 + 2t$ (s in metres, t in seconds)

(a) Find v & a as a f$^n$ of time.

(b) Find acceleration at 3 sec.

(a) $v = s'$

so

$v = 3t^2 + 4t + 2$

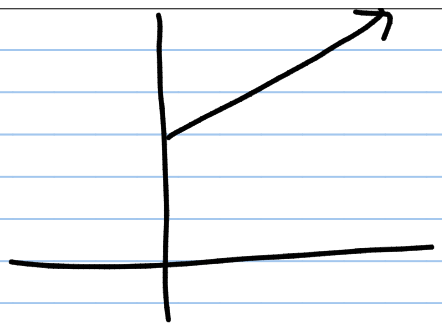
$a = s''$

$a = 6t + 4$

b) $a(3) = 6(3) + 4$

$= 22 \text{ m/s}^2$

$d = 5t + 6$



$v = 5\ m/s$

~~test~~ $v(2) = 5\ m/s$

$v(7) = 5\ m/s$

$a = 0\ m/s^2$

Now you can do

3.1 & 3.2