3.3 Rates of Change in the Natural Sciences

Recall: If y is a quantity that depends on another quantity, x, then:

$$\Delta y = f(x_2) - f(x_1) \quad \text{as} \quad x_1 \to x_2$$

The Instantaneous rate of change as $x_2 \to x_1$ is: $f'(x_1)$

Example

Find the **RATE OF CHANGE** of the Volume of a spherical Balloon w.r.t. the radius when $r = 10\text{ cm}$.

$\frac{dV}{dr}$ or V'

$$V(r) = \frac{4\pi r^3}{3}$$

$$V' = 4\pi r^2$$

$$V'(10) = 4\pi (10\text{ cm})^2$$

volume wrt radius

$$= 400\pi\ \text{cm}^3/\text{cm}$$

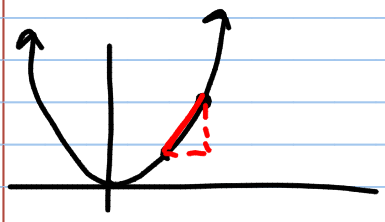
$$\approx 1257\ \text{cm}^3/\text{cm}$$

Ex 2 The mass of the left-hand x-metres of a metal rod is given by $f(x) = x^2$ kg.

(a) Find the average density of the part of the rod $2 \le x \le 2.3$ m.

Ch 1: $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$ $\dfrac{\Delta y}{\Delta x}$ $= \dfrac{(2.3)^2 - (2)^2}{2.3 - 2}$

$= \dfrac{1.29}{.3} = 4.3$ kg/m



(b) Find the linear density at $x = 2$ m (like a limit.... derivative!!)

$f'(x) = 2x$

$f'(2) = 2(2) = 4$ kg/m

Now do section 3.3