3.5 Related Rates

- In a RELATED RATES problem we are given the rate of change of one quantity and asked to find the rate of change of a related quantity
- To accomplish this we need a formula that relates the two quantities and then differentiate wrt time

For example: If the volume of a sphere is decreasing at certain rate, $\frac{dV}{dt}$ how fast is the radius decreasing?

$V = \frac{4\pi r^3}{3}$ Given: $\frac{dV}{dt}$ Asked for: $\frac{dr}{dt}$

HINTS Pg 143 → Problem solving ideas

(1) What are you given? What are you looking for?

Know: Want:

(2) Can you find a formula to relate the quantities?

(3) Draw a picture!

(4) RTFQ2X → make sure you know what they're asking!

(5) UNITS COUNT!

Ex 1 If $xy^2 = 12$ and $\frac{dy}{dt} = 6$, find $\frac{dx}{dt}$ when $y = 2$ (Want: $\frac{dx}{dt}$)

$f \cdot g' + f' \cdot g$

$$\frac{d}{dt}(xy^2) = \frac{d}{dt}(12)$$

$$x \cdot 2y \cdot \frac{dy}{dt} + 1 \cdot \frac{dx}{dt} \cdot y^2 = 0$$

(want: $\frac{dx}{dt}$)

$$\frac{dx}{dt} \cdot y^2 = -2xy \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{-2xy \cdot dy/dt}{y^2} = \frac{-2(3) \cdot 6}{2} = -18$$

If $x \cdot y^2 = 12$ → when $y = 2$

$x \cdot 4 = 12$

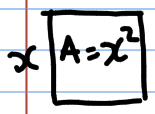
$x = 3$

-18 unit/time

Ex #2 How fast is the area of a square increasing when the side is 3 m in length and growing at a rate of 0.8 m/min?

Know: when $x = 3m$

Want: $\frac{dA}{dt}$ $\left(\frac{m^2}{min}\right)$



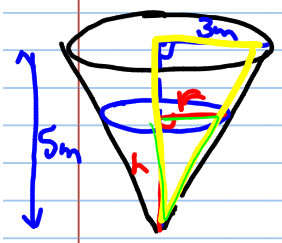
$A = x^2$

$\frac{dx}{dt} = 0.8\frac{m}{min}$

$\frac{dA}{dt} = 2x \cdot \frac{dx}{dt}$

$\frac{dA}{dt} = 2 \cdot 3m \cdot .8\frac{m}{min} = 4.8\frac{m^2}{min}$

#3 A conical water tank is 5m high and has a diameter of 6m at the top. Water is being pumped into the tank at a rate of $1.6\ m^3/min$. Find the rate at which the water level is rising when the water is 2m deep.



How do r + h relate?

Know:
- cone-shaped $r=3,\ h=5$
- $h=2m$
- $dV/dt = 1.6\ m^3/min$

Want:
- $\frac{dh}{dt}$

$\frac{r}{h} = \frac{3}{5}$

$r = \frac{3h}{5}$

Formula: $V = \frac{\pi r^2 h}{3} = \frac{\pi\left(\frac{3h}{5}\right)^2 \cdot h}{3}$

$V = \frac{3\pi h^3}{25}$

$\frac{dV}{dt} = 3 \cdot \frac{3\pi}{25} \cdot h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{9\pi}{25} h^2 \cdot \frac{dh}{dt}$

$\downarrow$

$1.6\frac{m^3}{min} = \frac{9\pi}{25}(2m)^2 \cdot \frac{dh}{dt}$

$\frac{1.6\frac{m^3}{min}}{\frac{9\pi}{25} \cdot 4\,m^2} = \frac{10}{9\pi}\frac{m}{min}$

or $0.3537\ m/min$