Derivatives of Trig Functions

$$\frac{d}{d\theta}(\sin\theta) = \cos\theta$$

... really means:

$$\frac{d}{d\theta}(\sin\theta) = \cos(\theta)\left(\frac{d}{d\theta}(\theta)\right)$$

or

If $f(\theta) = \sin\theta$

then $f'(\theta) = \cos(\theta)\cdot\theta'$

The Chain Rule strikes again!!

Ex: $y = \sin x$

$y' = \cos(x)\cdot 1 = \cos x$

$y = \sin(x^2)$

$y' = \cos(x^2)\,2x = 2x\cos x^2$

To prove the following, feel free to use:

$$f' = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$\frac{d}{dx} \sin x = \cos x$ $\quad$ $\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$ $\quad$ $\frac{d}{dx} \csc x = -\csc x \cdot \cot x$

$\frac{d}{dx} \sec x = \sec x \tan x$ $\quad$ $\frac{d}{dx} \cot x = -\csc^2 x$

Examples:

(a) $y = \tan(3x^2+4)$

$y' = \sec^2(3x^2+4) \cdot 6x$ ←

$y' = 6x \sec^2(3x^2+4)$

(b) $y = \sin(x^3)$

$y' = 3x^2 \cos(x^3)$

(c) $y = \sin^3 x = (\sin x)^3$

$y' = 3(\sin x)^2 \cdot \cos x$

(d) $y = \sin^3(x^2-1)$ (means $(\sin(x^2-1))^3$)

$y' = 3(\sin(x^2-1))^2 \cdot (\cos(x^2-1)) \cdot (2x)$

$\Rightarrow y' = 6x(\sin(x^2-1))^2 \cos(x^2-1)$

(e) $y = x^2 \cdot \sec(3x)$

$y' = x^2 \cdot \sec(3x)\tan(3x)\cdot 3 + 2x \cdot \sec 3x$

$y' = 3x^2(\sec 3x \tan 3x) + \sec(3x)\, 2x$

$x(\sec 3x)[3x\tan 3x + 2]$ ★

(f) $\frac{d}{dx}(\sin x + \sin y = 1)$ find $\frac{dy}{dx}$ or y'

$\cos x + \cos y \cdot y' = 0$

$-\cos x$ $-\cos x$

$\frac{\cos y \cdot y'}{\cos y} = \frac{-\cos x}{\cos y}$

$$y' = -\frac{\cos x}{\cos y}$$

(1) Find the EQUATION of the tangent line to $y = \frac{\sin x}{\cos 2x}$ where $x = \frac{\pi}{6}$

(Exact Answers only)

(2) If $f(x) = \frac{1}{1+\tan x}$ $(\tan x \neq -1)$

find $f'(x)$

(3) $y = 2\csc^3(3x^2)$ Find $f'(x)$

(4) $\tan y = x^2$ Find y'

(1) $y = \frac{\sin x}{\cos 2x}$ where $x = \frac{\pi}{6}$

$$y' = \frac{\cos(2x)\cdot\cos x - \sin x \cdot 2 \cdot (-\sin 2x)}{(\cos 2x)^2}$$

$$y' = \frac{\cos 2x \cdot \cos x + 2\sin x \sin 2x}{(\cos 2x)^2}$$

$$y'\left(\frac{\pi}{6}\right) = \frac{\cos 2\left(\frac{\pi}{6}\right)\cdot\cos\frac{\pi}{6} + 2\sin\frac{\pi}{6}\sin 2\cdot\frac{\pi}{6}}{\left(\cos 2\cdot\frac{\pi}{6}\right)^2}$$

$$= \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + 2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}}{\frac{1}{4}}$$

$$m = \frac{\frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4}}{\frac{1}{4}} = \frac{3\sqrt{3}}{4}\cdot\frac{4}{1} = 3\sqrt{3}$$

$x = \frac{\pi}{6}$

$$y - y_1 = m(x - x_1)$$

y_1: $y = \frac{\sin x}{\cos 2x}$

$$y = \frac{\sin\pi/6}{\cos\pi/3} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$y - 1 = 3\sqrt{3}\left(x - \frac{\pi}{6}\right)$$

$$f(x) = \frac{1}{1+\tan x} = (1+\tan x)^{-1}$$

$$f'(x) = -1(1+\tan x)^{-2} \cdot \sec^2 x$$

$$= -\frac{\sec^2 x}{(1+\tan x)^2}$$

$$y = 2(\csc(3x^2))^3$$

$$y' = 6(\csc(3x^2))^2 \cdot (-\csc(3x^2)\cot(3x^2)) \cdot 6x$$

$$= -36x \cdot \csc^3(3x^2) \cdot \cot(3x^2)$$

$\tan y = x^2$ find $\frac{dy}{dx}$

$\downarrow$

$\sec^2 y \cdot y' = 2x$

$y' = \frac{2x}{\sec^2 y}$ or $2x\cos^2 y$

7.2 $\frac{1}{\xi}$ 7.3

7.5/7.6