

ANTIDERIVATIVES

A function  $F$  is an antiderivative of  $f$  if  $F'(x) = f(x)$  (for all  $x$  on a given interval)

For Example:

If  $f(x) = 2x$      $F(x) = x^2$

since  $F'(x) = 2x = f(x)$

$f(x) = \frac{1}{2}x^{-1/2}$      $F(x) = \sqrt{x}$  or  $x^{1/2} + C$

$f(x) = \frac{3}{5}x^2$      $F(x) = \frac{1}{5}x^3 + C$

$f(x) = x^6$      $F(x) = \frac{1}{7}x^7 + C$

If  $f(x) = x^n$ ,  $F(x) = \frac{x^{n+1}}{n+1} + C$

$f(x) = 4x^3 - 6x^2 + 11$

$F(x) = x^4 - 2x^3 + 11x + C$

Check:  $F'(x) = 4x^3 - 6x^2 + 11 = f(x)$  ✓

$f(x)$	$F(x)$
0	$C$
1	$x + C$
$x^n$	$\frac{x^{n+1}}{n+1} + C$
<u><math>e^{kx}</math></u>	$\frac{1}{k}e^{kx} + C$
$\cos kx$	$\frac{1}{k}\sin(kx) + C$
$\sin kx$	$-\frac{1}{k}\cos kx + C$
$2x^2 - x + 7$	$\frac{2x^3}{3} - \frac{1}{2}x^2 + 7x + C$
$\frac{2}{x^2} - \frac{5}{x} + x$	$\frac{2x^{-1}}{-1} - 5 \cdot \ln x  + \frac{1}{2}x^2 + C$
$2x^{-2} - 5\left(\frac{1}{x}\right) + x$	$= -\frac{2}{x} - 5\ln x  + \frac{1}{2}x^2 + C$
$2^x \ln 2$	$2^x + C$
$f(x) = \sin x \cos x$	$\frac{1}{2}\sin^2 x + C$
$\cos x \cdot \sin x$	$-\frac{1}{2}\cos^2 x + C$

How do we find 'C'?

$\Rightarrow$  to find C, we need some more info.

Ex: Find  $F(x)$  if  $f(x) = 2x$   
when  $F(x) = \underline{\underline{3}}$  and  $x = \underline{\underline{0}}$

$$F(x) = x^2 + C$$

$\downarrow$

$$\underline{\underline{3}} = (\underline{\underline{0}})^2 + C$$

$$C = 3$$

so:  $F(x) = x^2 + 3$

Now do  
9.1 & 9.2