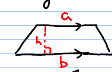
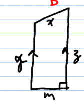
Ch 10 Pre-Review

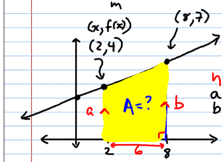
Area of a Trapezoid



$A = \frac{1}{2}h(a+b)$



$A = \frac{1}{2}(m)(y+z)$



$h=6$
$a=4$
$b=7$

$A=\frac{1}{2}(6)(4+7)$
$= 3(11)$
$= 33 \text{ units}^2$

Sigma Notation

$$\sum_{i=1}^{n} t_i = t_1 + t_2 + t_3 + \ldots + t_n$$

Example:

$$\sum_{i=3}^{7} i^2+1 = [3^2+1] + [4^2+1] + [5^2+1] + [6^2+1] + [7^2+1]$$

$$= 5+9+16+25+36+49$$

$$= \underline{\underline{140}}$$

some things to note:

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c \text{ n times}$$

or $= cn$

$$\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i$$

$$\sum_{i=1}^{n} 7.2(i^2-3) = 7.2\left[\sum_{i=1}^{n}(i^2-3)\right]$$

$$\sum_{i=1}^{n} [t_i + q_i] = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} q_i$$

Ex: $\sum_{i=1}^{n} (2i+4)^2$

$$\sum_{i=1}^{n} (4i^2+16i+16)$$

$$\sum_{i=1}^{n} 4i^2 + \sum_{i=1}^{n} 16i + \sum_{i=1}^{n} 16$$

$$4\sum_{i=1}^{n} i^2 + 16\boxed{\sum_{i=1}^{n} i} + 16n$$

Hey Guess what!

$$\sum_{i=1}^{n} i = \boxed{\frac{n(n+1)}{2}}$$

Comes from:

☆ $\sum_{i=1}^{n} i = 1+2+3+4+5+\ldots+n$

$t_1 = 1$
$d = 1$
$n = n$

$S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{n}{2}[2(1) + (n-1)1]$

$= \frac{n}{2}[2+n-1]$

$= \frac{n}{2}[n+1]$

☆ $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

☆ $$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

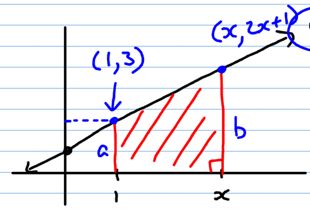
$$4\sum_{i=1}^{n} i^2 + 16\sum_{i=1}^{n} i + 16n$$

$$= \cancel{4}^{2}\left(\frac{n(n+1)(2n+1)}{\cancel{6}3}\right) + \cancel{16}^{8}\left[\frac{n(n+1)}{\cancel{2}}\right] + 16n$$

$$= \frac{2n(n+1)(2n+1)}{3} + 8n(n+1) + 16n$$

...

10.1 Area Under Curve



$y = 2x+1$

$A = \frac{1}{2}h(a+b)$

$h = x - 1$
$a = ? = 3$
$b = ? = (2x+1)$

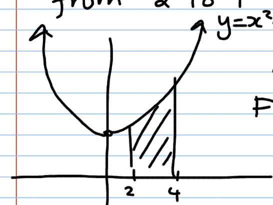
$$A = \frac{1}{2}(x-1)(3+2x+1)$$
$$= \frac{1}{2}(x-1)(2x+4)$$
$$= \frac{1}{\cancel{2}}(x-1)(\cancel{2})(x+2)$$
$$A(x) = x^2 + x - 2$$

$$A'(x) = 2x+1$$

So: The Anti-derivative of the function is the Area Function!

In general:
If F is an antiderivative of the positive function f, then the area under $y = f(x)$ from a to b is:
$$A = F(b) - F(a)$$

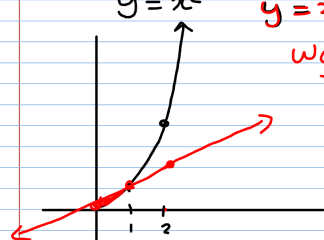
Find the area under $y = x^2+2$ from 2 to 4



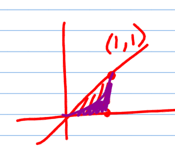
$$A = F(4) - F(2)$$
$$F(x) = \frac{1}{3}x^3 + 2x + C$$

$$A = \left(\frac{1}{3}(4)^3 + 2(4) + C\right) - \left(\frac{1}{3}(2)^3 + 2(2) + C\right)$$
$$= \frac{64}{3} + 8 \cancel{+C} - \frac{8}{3} - 4 \cancel{-C}$$
$$= \frac{56}{3} + 4 = \frac{56}{3} + \frac{12}{3} = \frac{68}{3}$$

$y = x^2$ $y = x$



Want the area between the curves!



A = Area under the top curve − Area under the bottom curve.

Now do Ch 10 PreReview
10.1
10.2