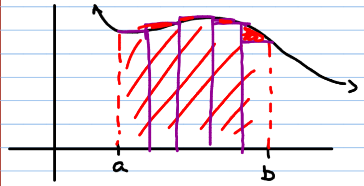
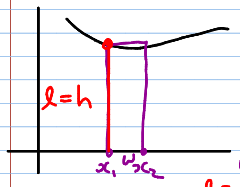
Area as Limits



- Adding up the area of individual rectangles would give us an approx. area under the curve
- by increasing the number of rectangles by making them skinnier, gives us a better approximation
- Letting the width of the rectangles →0, gives us an Exact area

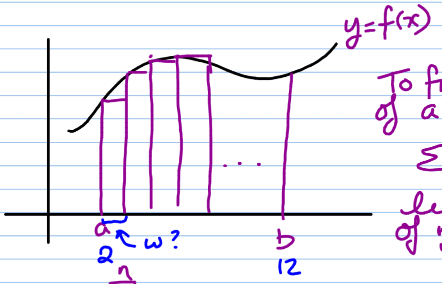
$A = lw$



$w = x_2 - x_1$

$l = h = f(x_1)$

$A = f(x_1)[x_2 - x_1]$



To find the sum of all the areas:

$\sum A$

Let the number of rectangles be "n"

$\sum_{i=1}^{n} A$

$w = \frac{b-a}{n} = \Delta x$ $\qquad$ $h = l = f(x_i)$

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \overset{l}{f(x_i)} \cdot \underset{\uparrow\, \frac{b-a}{n}}{\overset{w}{\Delta x}}$$

Then: Let f be a CONTINUOUS function defined on an interval [a,b]. Then The DEFINITE INTEGRAL of f from a to b is:

upper limit $\qquad$ integrand

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Lower Limit $\qquad$ we'll come back to later!

where $\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

If $f(x) \geq 0$, then

$\int_a^b f(x)dx$ = area under the curve ⇝ $\int$ = anti-deriv.

Ex: Evaluate $\int_0^5 (3x - x^2)\,dx$

The quick way:

$$A = \frac{3}{2}x^2 - \frac{1}{3}x^3 + C\Big]_0^5$$

$$= \left(\frac{3}{2}(5)^2 - \frac{1}{3}(5)^3 + C\right) - \left(\frac{3}{2}(0)^2 - \frac{1}{3}(0)^3 + C\right)$$

$$\boxed{= \frac{75}{2} - \frac{125}{3} = \frac{-25}{6}}$$

$$\frac{225}{6} - \frac{250}{6} = \uparrow$$

Now using limits!

$$\int_0^5 (3x - x^2)\,dx \qquad \Delta x = \frac{5-0}{n} = \frac{5}{n}$$

$$\underline{\underline{x_i}} = 0 + i \cdot \frac{5}{n} = \frac{5i}{n}$$

$$\int_0^5 \overset{f(x)}{(3x - x^2)}\,dx = \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{5i}{n}\right) \cdot \left(\frac{5}{n}\right)}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \overset{f\left(\frac{5i}{n}\right)}{\left[3\left(\frac{5i}{n}\right) - \left(\frac{5i}{n}\right)^2\right]} \left[\frac{5}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{15i}{n} - \frac{25i^2}{n^2}\right]\left[\frac{5}{n}\right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{75i}{\underline{\underline{n^2}}} - \frac{125i^2}{\underline{\underline{n^3}}}\right]$$

$$\lim_{n\to\infty} \left[\frac{75}{n^2} \boxed{\sum_{i=1}^{n} i} - \frac{125}{n^3} \boxed{\sum_{i=1}^{n} i^2}\right]$$

$$\lim_{n\to\infty} \left[\frac{75}{n^2}\left[\frac{n(n+1)}{2}\right] - \frac{125}{n^3}\left[\frac{\overset{(n^2+n)}{n(n+1)}(2n+1)}{6}\right]\right]$$

$$= \lim_{n\to\infty} \left[\frac{75}{2}\left[\frac{n^2+n}{n^2}\right] - \frac{125}{6}\left[\frac{2n^3+3n^2+n}{n^3 n^3 \quad n^3}\right]\right]$$

$$= \lim_{n\to\infty} \left[\frac{75}{2}\left[1 + \overset{0}{\cancel{\frac{1}{n}}}\right] - \frac{125}{6}\left[2 + \overset{0}{\cancel{\frac{3}{n}}} + \overset{0}{\cancel{\frac{1}{n^2}}}\right]\right]$$

$$= \frac{75}{2} - \frac{250}{6}$$

$$= \frac{75}{2} - \frac{125}{3} = \boxed{\frac{-25}{6}}$$

Assignment

Ch 11 PRE-REVIEW.