Calculus 12 June $5^{th}$

(1) Partial Fractions Method
(2) Worksheets
(3) Mock Final

Time Line:

WED: Partial Fractions
THUES: Partial Fractions
FRI: REVIEW/Worksheets
MON: REVIEW/WORKSHEETS
TUES: CH 9-11 TEST
WED:
THURS:
FRI:
REVIEW for Final

FINAL EXAM: TUES JUNE $18^{th}$

12:30 - 2:30

(But you can have until 3:30)

# 11.6 Partial Fractions

- Reverse of the Quotient Rule!

Our Goal is to express a single fraction as the sum of two or more other fractions.

## Type I Questions:

- Denominator is the product of distinct Linear factors

Ex $\displaystyle\int \frac{5x+1}{(x-1)(x+2)}$ or $\displaystyle\int \frac{7x+6}{(x-3)(2x+5)}$

Ex 1 $\displaystyle\int \frac{(2x-1)\,dx}{x^2-16} = \int \frac{(2x-1)}{(x-4)(x+4)}\,dx$

Partial fractions:

$$\frac{(2x-1)\cancel{(x-4)}\cancel{(x+4)}}{\cancel{(x-4)}\cancel{(x+4)}} = \frac{A\cancel{(x-4)}(x+4)}{\cancel{(x-4)}} + \frac{B(x-4)\cancel{(x+4)}}{\cancel{(x+4)}}$$

① Mult. by a CD

$2x-1 = A(x+4) + B(x-4)$

② Group the "x" terms

$2x-1 = Ax + 4A + Bx - 4B$

$2x-1 = Ax + Bx + (4A - 4B)$

$x - \;\; = (\;\;)x - (\;\;)$

$(A+B = 2)\times 4 \rightarrow 4A + 4B = 8$

$-4A + 4B = 1 \rightarrow -4A + 4B = 1$

$8B = 9$

$B = 9/8$

$A + B = 2$

$A = 7/8$

SO: $\displaystyle\frac{2x-1}{(x-4)(x+4)} = \frac{A}{(x-4)} + \frac{B}{(x+4}$

$\displaystyle = \frac{7/8}{x-4} + \frac{9/8}{x+4}$

$$\int \frac{2x-1\,dx}{(x-4)(x+4)} = \int \frac{7/8\,dx}{x-4} + \int \frac{9/8\,dx}{x+4}$$

$$= \frac{7}{8}\ln|x-4| + \frac{9}{8}\ln|x+4| + C$$

Type II: Denominator is the Product of linear factors, some of which are repeated.

$$\int \frac{2x+3}{(x+2)^2(x+1)}\,dx \quad \text{or} \quad \int \frac{3x\,dx}{(x-1)^4(x+3)^3}$$

Ex 1 $\int \frac{(6x+7)dx}{(x+2)^2}$

$$\frac{(6x+7)}{\cancel{(x+2)^2}}\cancel{(x+2)^2} = \frac{A}{\cancel{(x+2)}}(x+2)^2 + \frac{B}{\cancel{(x+2)^2}}\cancel{(x+2)^2}$$

$$6x+7 = A(x+2) + B$$

$$6x+7 = Ax + (2A+B)$$

$$A=6$$

$$2A+B=7$$
$$12+B=7$$
$$B=-5$$

subst.

$$\int \frac{(6x+7)dx}{(x+2)^2} = \int \frac{6\,dx}{(x+2)} + \int \frac{-5\,dx}{(x+2)^2}$$

$(x+2)^{-2}$

$$= 6\cdot \ln|x+2| - 5\frac{(x+2)^{-1}}{-1} + C$$

$$= 6\ln|x+2| + \frac{5}{x+2} + C$$

Type III - denominator includes at least one non-repeated, irreducible quadratic factor

Ex: $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$

$$\frac{(-2x+4)}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$(Ax+B)(x-1)(x-1)$

$$-2x+4 = (Ax+B)(x-1)^2 + C(x^2+1)(x-1) + D(x^2+1)$$

FOIL, EXPAND, SIMPLIFY and collect or Group like terms.

$$-2x+4 = (A+C)x^3 + (-2A+B-C+D)x^2 + (A-2B+C)x + (B-C+D)$$

$A+C=0$

$-2A+B-C+D=0$

$A-2B+C=-2$

$B-C+D=4$

$-2A+(B-C+D)=0$

$B-C+D=4$

$-2A+4=0$

$-2A=-4$

$$\boxed{\begin{array}{c} A=2 \\ \therefore\ C=-2 \\ B=1 \\ \text{and } D=1 \end{array}}$$

(from $A+C=0$)

$\cancel{2}-2B+(\cancel{-2})=-2$

$B=1$

So:

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx = \int \frac{2x+1}{x^2+1}\,dx + \int \frac{-2\,dx}{x-1} + \int \frac{1}{(x-1)^2}\,dx$$

$$= \boxed{\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}}\,dx - 2\int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2}$$

$$= \ln|x^2+1| + \tan^{-1}x - 2\ln|x-1| + \frac{(x-1)^{-1}}{-1} + C$$

$$\int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u} = \ln|u|$$

$u = x^2+1$ $\qquad = \ln|x^2+1|$

$du = 2x\,dx$

TypeIV: The degree of the numerator is higher than the denominator → you <u>DIVIDE</u> First & then have a remainder...

$$\int \frac{x^4+3x^3+2x+1}{x^3+1} = \int (x+3) + \frac{x-2}{x^3+1}$$

$$\begin{array}{r l}
 & \quad x + 3 \\
x^3+1 \,) & \cancel{x^4} + 3x^3 + 2x + 1 \\
 & \cancel{x^4} + \qquad\quad + x \\
\hline
 & \qquad 3x^3 + x + 1 \\
 & \qquad 3x^3 \qquad + 3 \\
\hline
 & \qquad\qquad x - 2
\end{array}$$

$$\begin{array}{r l}
 & 1\ \cancel{2}3 \\
10 \,) & 113 \\
 & 10 \\
\hline
 & \ \ 3
\end{array}$$

$$1 + \frac{3}{10}$$

# Section 11.6 Pg 525

1–17 ODDS (omit 15).

for 1–7 set it up, DO NOT solve! (i.e., read the instructions!!)