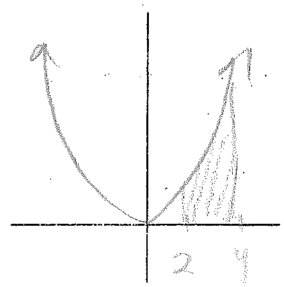
Area Under a Curve Quiz

Name:________________

Find the area under each curve between the indicated limits. (4 marks)
Draw a sketch of each area. (1 mark)

1. $y = x^2$ from 2 to 4.



$A'(x) = x^2$

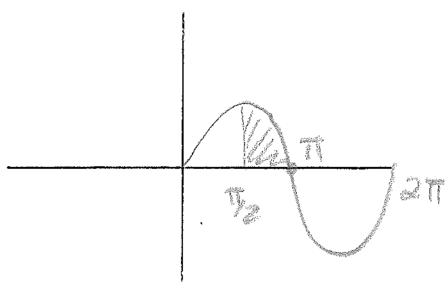
$F(x) = \frac{1}{3}x^3$

$A(4) = F(4) - F(2)$

$= \frac{1}{3}4^3 - \frac{1}{3}2^3$

$= \frac{56}{3} = \underline{18\,2/3}$

2. $y = \sin x$ from $\frac{\pi}{2}$ to π.



$A'(x) = \sin x$

$F(x) = -\cos x$

$A(\pi) = F(\pi) - F(\pi/2)$

$= -\cos\pi - \left(-\cos\frac{\pi}{2}\right)$

$= -(-1) + 0$

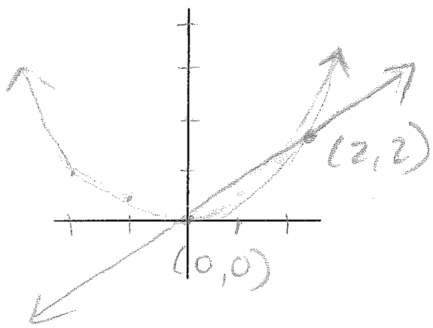
$= \underline{1}$

Score: /10

Area Between Curves Quiz Name:________________

Find the area between the two curves given. (4 marks)
Draw a sketch of each area. (1 mark)

1. Area between $y = \frac{x^2}{2}$ and $y = x$.



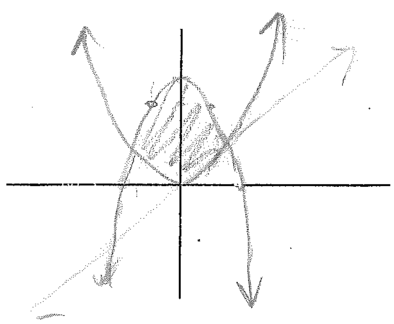
$A'(x) = x - \frac{x^2}{2}$

$F(x) = \frac{1}{2}x^2 - \frac{1}{6}x^3$

$A(2) = \left[\frac{1}{2}(2)^2 - \frac{1}{6}2^3\right] - [0 - 0]$

$= 2 - \frac{8}{6} = \frac{2}{3}$

2. Area between $y = -x^2 + 4$ and $y = x^2$.



$-x^2 + 4 = x^2$

$2x^2 - 4 = 0$

$2x^2 = 4$

$x^2 = 2$

$x = \pm\sqrt{2}$

$A'(x) = (x^2+4) - x^2 = -2x^2 + 4$

$F(x) = -\frac{2}{3}x^3 + 4x$

$\left[-\frac{2}{3}(\sqrt{2})^3 + 4\sqrt{2}\right] - \left[-\frac{2}{3}(-\sqrt{2})^3 + 4(-\sqrt{2})\right]$

Score: /10

$= -\frac{4\sqrt{2}}{3} + 4\sqrt{2} - \left[\frac{4\sqrt{2}}{3} - 4\sqrt{2}\right] = -\frac{8\sqrt{2}}{3} + \frac{8\sqrt{2}}{3} = \frac{16\sqrt{2}}{3}$

7.54

EVALUATING DEFINITE INTEGRALS
Calculus 12

Name: Key

Evaluate the following definite integrals:

1. $\int_{-3}^{5} x^2 dx = \frac{1}{3}x^3\Big]_{-3}^{5} = \frac{125}{3} + \frac{27}{3} = \frac{152}{3}$ /2

2. $\int_{1}^{4} (x^3 - 3x^2 + 5x + 1)dx = \frac{x^4}{4} - x^3 + \frac{5}{2}x^2 + x\Big]_{1}^{4} =$

$44 - \frac{11}{4} = 41\frac{1}{4}$ or $\frac{165}{4}$ /2

☆ 3. $\int_{0}^{2\pi} (\sin x + \cos x)dx$

$= -\cos x + \sin x\Big]_{0}^{2\pi} = [-1+0] - [-1+0] = 0$ /2

4. $\int_{0}^{2} (e^x + \frac{1}{x})dx = e^x + \ln x\Big]_{0}^{2}$ UNDEFINED /1

5. $\int_{-3}^{2} (\frac{1}{x^2} - 2)dx$ Is not continuous on the interval [-3, 2] therefore the integral is undefined. /1

/8

/18

INTEGRATION BY SUBSTITUTION

Calculus 12

Name: Key.

Integrate the following using substitution. Clearly state u and du.

1. $\int\sqrt{4x-1}dx$ $\quad u = 4x-1 \quad du = 4dx.$

$$\int u^{1/2}\frac{du}{4} = \frac{1}{4}\left(\frac{2}{3}u^{3/2}\right)+C$$

$$= \frac{1}{6}(4x-1)^{3/2} + C. \checkmark$$

/3

2. $\int\cos(7x+5)dx$ $\quad u = 7x+5 \quad du = 7dx.$

$$\frac{1}{7}\int\cos u\,du = \frac{1}{7}\sin u + C$$

$$= \frac{1}{7}\sin(7x+5) + C. \checkmark$$

/3

3. $\int\frac{1}{\cos^2 2x}dx = \int\sec^2 2x\,dx$ $\quad u = 2x \quad du = 2dx.$

$$= \int\sec^2 u\cdot\frac{1}{2}du$$

$$= \frac{1}{2}\tan u + C$$

$$= \frac{1}{2}\tan 2x + C. \checkmark$$

/3

4. $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ $\quad u = \cos x \quad du = -\sin x\,dx$

$$= -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|\cos x| + C.$$

or $\ln|\sec x| + C$ ✓

5. $\int (x^2+2x-3)^2(x+1)\,dx$ $\quad u = x^2 + 2x - 3 \quad du = (2x+2)\,dx$
$= 2(x+1)\,dx.$

$$\int u^2 \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2} \cdot \frac{1}{3} u^3 + C$$

$$= \frac{1}{6}(x^2+2x-3)^3 + C.$$ ✓

6. $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x\,dx$ $\quad u = \tan x \quad du = \sec^2 x\,dx.$

$$= \int_0^{\cancel{\pi/4}\,1} u\,du = \frac{1}{2}u^2\Big]_0^1 = \underline{\underline{\frac{1}{2}}}$$ ✓

INTEGRATION BY PARTS
Calculus 12

$\frac{}{20}$

Name: Key

Integrate the following using integration by parts. Clearly state u, du, v and dv.

1: $\int x^2 e^x dx$

① $u = x^2$ $\quad dV = e^x dx$
$du = 2x\,dx$ $\quad V = e^x$

$$\overset{①}{=} x^2e^x - \int e^x \cdot 2x\,dx$$

② $u = 2x$ $\quad dV = e^x dx$
$du = 2\,dx$ $\quad V = e^x$

$$= x^2 e^x - \overset{②}{\left[2xe^x - \int e^x 2\,dx\right]} = \underline{\underline{x^2e^x - 2xe^x + 2e^x + C}}$$

/3

$u = x$ $\quad dV = \cos x\,dx$
$du = dx$ $\quad V = \sin x$

2. $\int x \cos x dx$ $= x\sin x - \int \sin x\,dx$

$$= \underline{\underline{x \sin x + \cos x + C}}$$

/3

~~$u = \cos x$~~
~~$du = (-\sin x)dx$~~

① $u = e^x$ $\quad dV = \cos x\,dx$
$du = e^x dx$ $\quad V = \sin x$

② $u = e^x$ $\quad dV = \sin x$
$du = e^x dx$ $\quad V = -\cos x$

3. $\int e^x \cos x dx$ $\overset{①}{=} e^x \sin x - \int \sin x\, e^x dx$

$$= e^x \sin x - \overset{②}{\left[-e^x\cos x + \int \cos x\, e^x dx\right]}$$

/4

$$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int \cos x\, e^x \cancel{dx}$$
$$+ \int e^x \cos x\,dx \qquad\qquad + \int \cancel{\cos x}\, e^x dx$$

$$\frac{2}{2}\int e^x \cos x\,dx = \frac{e^x \sin x + e^x \cos x}{2}$$

$$= \frac{e^x}{2}(\sin x + \cos x) + C$$

/10

① $u = x^2 - 5x$ $\quad dV = e^x dx$ ‖ ② $u = 2x - 5$ $\quad dV = e^x dx$

$du = (2x-5)dx$ $\quad V = e^x$ ‖ $du = 2dx$ $\quad V = e^x$

4. $\int (x^2 - 5x)e^x dx = \overset{①}{(x^2-5x)e^x} - \overset{②}{\int e^x(2x-5)dx}$

$= (x^2-5x)e^x - e^x(2x-5) + \int e^x 2dx$

$= (x^2-5x)e^x - (2x-5)e^x + 2e^x + C$

$= e^x(x^2 - 5x - 2x + 5 + 2) + C$

$= e^x(x^2 - 7x + 7) + C$

/3

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5. $\int e^{2x}\cos 3x dx = \overset{①}{\frac{1}{3}e^{2x}\sin 3x} - \overset{②}{\int \frac{1}{3}\sin 3x \cdot 2e^{2x}dx}$

① $u = e^{2x}$
$du = 2e^{2x}dx$
$dV = \cos 3x dx$
$V = \frac{1}{3}\sin 3x$

② $u = e^{2x}$
$du = 2e^{2x}dx$
$dV = \sin 3x dx$
$V = -\frac{1}{3}\cos 3x$

$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[e^{2x}\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)2e^{2x}dx\right]$

$= \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int \cos 3x e^{2x}dx$

$+\frac{4}{9}\int\cos 3x e$

$+\frac{4}{9}\int \cos 3x e^{2x}dx$

$\frac{13}{9}\int \cos 3x e^{2x}dx = \frac{\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x}{13/9}$

$\frac{\frac{13}{9}}{\frac{13}{9}}$

$\int \cos 3x e^{2x}dx = \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x) + C$

or

/4

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6. $\int_0^3 xe^{-x}dx$

$u = x \quad dV = e^{-x}dx$

$du = dx \quad V = -e^{-x}$

$= -xe^{-x}\Big]_0^3 - \int_0^3 -e^{-x}dx$

$= -xe^{-x}\Big]_0^3 + -e^{-x}\Big]_0^3$

$= [-3e^{-3} - 0] + [-e^{-3} - (-e^0)]$

$= -3e^{-3} - e^{-3} + 1$

$= -4e^{-3} + 1$ or $1 - \frac{4}{e^3}$

$\approx .80$

/3

10

/14

PARTIAL FRACTIONS
Calculus 12

Name: Key.

Write the following as Partial Fractions – no need to integrate.

1. $\frac{1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$ $\boxed{= \frac{1}{x+2} - \frac{1}{x+3}.}$

$$1 = Ax + 3A + Bx + 2B$$

$$1 = (A+B)x + (3A+2B)$$

$$3A+3B=0$$

$$3A+2B=1$$

$$B=-1$$

so $A = 1$

/4

2. $\frac{6x+7}{(x+2)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{6}{x+2} - \frac{5}{(x+2)^2}$

$$6x+7 = Ax + 2A + B$$

$$A = 6$$

$$2A + B = 7$$

so $B = -5$

/4

3. $\frac{5x^3 - x^2}{x^2 - 1} = 5x - 1 + \frac{2}{x-1} + \frac{3}{x+1}$

$$
\begin{array}{r}
5x - 1 \\
x^2 - 1 \overline{)\, 5x^3 - x^2 } \\
\underline{5x^3 - 5x } \\
-x^2 + 5x \\
\underline{-x^2 + 1} \\
+5x - 1
\end{array}
$$

$$\frac{5x^3 - x^2}{x^2 - 1} = 5x - 1 + \frac{5x - 1}{(x-1)(x+1)}$$

$$\frac{5x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$5x - 1 = Ax + A + Bx - B$

$5x - 1 = (A + B)x + (A - B)$

$A + B = 5$

$A - B = -1$

$2A = 4$

$A = 2$

so $B = 3$

/4

Volumes of Revolution Quiz Name: Key

Each question is worth 5 marks. Show all work.

1. Find the volume generated when the curve $y = x^2 + 3x + 2$ is rotated about the x-axis between the limits of $x = 4$. and $x = 7$.

$V = \pi \int_4^7 (x^2+3x+2)^2 dx = \pi \int_4^7 (x^4 + 3x^3 + 2x^2 + 3x^3 + 6x + 9x^2 + 2x^2 + 6x + 4) dx$

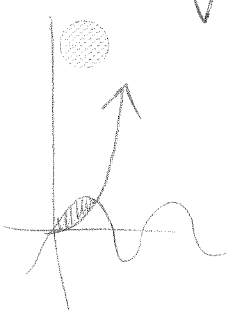
$= \pi \int_4^7 (x^4 + 6x^3 + 13x^2 + 12x + 4) dx = \pi \left(\frac{x^5}{5} + \frac{3x^4}{2} + \frac{13x^3}{3} + 6x^2 + 4x \right]_4^7$

$= \pi (8771.2\bar{3} - 978.1\bar{3})$

$= \underline{7793.1\pi}$

5/5

2. Find the volume of the solid generated by revolving the area bounded by the curves $y = \sin 3x$ and $y = x^2$ and the line ~~$y = 1.5$~~ $x = .75$, about the X-axis.



$V = \pi \int_0^{.75} (\sin^2 3x \, dx) - \pi \int_0^{.75} x^4 dx$

$= \frac{\pi}{2} \int_0^{.75} (1 - \cos 6x) dx - \frac{\pi x^5}{5} \Big]_0^{.75}$

$= \frac{\pi}{2} \left[x - \frac{1}{6} \sin 6x \right]_0^{.75} - \frac{\pi x^5}{5} \Big]_0^{.75}$

$= \frac{\pi}{2}(.913) - \pi(.047)$

$= \underline{1.286}$

5/5

3. Find the volume generated when the curve $y = e^x$ is rotated about the X-axis between the limits of 2 and 5.

$y = e^x$

$V = \pi \int_2^5 (e^x)^2 dx = \pi \frac{e^{2x}}{2} \Big]_2^5$

$= \pi \frac{e^{10}}{2} - \pi \frac{e^4}{2}$

$= \underline{34513.32899}$

or 10985.6π

5/5