Limit of a Function Quiz   Name: ANS KEY

Find the following limits: (if there is no limit, state "no limit")

① $\lim\limits_{x \to 4} x^2$ $\quad$ 16.

② $\lim\limits_{x \to 3} x^2 + 5x + 6$ $\quad 9 + 15 + 6 = 30$

③ $\lim\limits_{x \to 1} \dfrac{x-1}{x^2-1}$ $\quad \dfrac{1}{x+1}$ $\quad \dfrac{1}{2}$

④ $\lim\limits_{x \to -1} \dfrac{x-1}{x^2-1}$ $\quad \dfrac{1}{x+1}$ $\quad$ no limit

⑤ $\lim\limits_{x \to 2} \dfrac{x^3-8}{x^2-4}$ $\quad (x-2)\dfrac{x^2+2x+4}{x+2(x-2)} = \dfrac{4+4+4}{4} = \cancel{3}\ 3$

⑥ $\lim\limits_{h \to 3} \dfrac{(h-1)^2-1}{h}$ $\quad \dfrac{3}{3}$ $\quad \cancel{\dfrac{h^2+2h}{h}}$ $\quad \dfrac{h^2+2h+1-1}{h}$ $\quad \dfrac{h(h-2)}{h} = h-2 = 1.$

⑦ $\lim\limits_{x \to 0} \dfrac{(x-2)^2-4}{x}$ $\quad = \dfrac{x^2-4x+\cancel{4-4}}{x} = x-4 = -4.$

⑧ $\lim\limits_{h \to -2} \dfrac{\frac{1}{h}-\frac{1}{2}}{h+2}$ $\quad \dfrac{2-h}{2h(h+2)}$ $\quad$ no limit

⑨ $\lim\limits_{h \to -2} \dfrac{\frac{1}{h}+\frac{1}{2}}{h+2}$ $\quad \dfrac{\cancel{(2+h)}}{2h\cancel{(h+2)}} = \dfrac{1}{2h} = -\dfrac{1}{4}$

⑩ $\lim\limits_{h \to 0} \dfrac{\frac{1}{1+h}-1}{h}$ $\quad = \dfrac{1-(1+h)}{h(1+h)} = \dfrac{-\cancel{h}}{\cancel{h}(1+h)} = \dfrac{-1}{1+h} = -1$

/10

Limits Quiz Name: ANS KEY

Evaluate each of the following limits. Show all your work. (2 marks each)

1. $\lim_{x \to -2} \frac{x+7}{x^2+3x+7} = \frac{5}{4-6+7} = \frac{5}{5} = 1.$

2. $\lim_{x \to 5} \frac{x^2-8x+15}{x^2+2x-35} = \frac{\cancel{(x-5)}(x-3)}{\cancel{(x-5)}(x+7)} \quad \frac{2}{12} = \frac{1}{6}$

3. $\lim_{h \to 0} \frac{(3+h)^2-9}{h} = \frac{\cancel{9}+6h+h^2-\cancel{9}}{h} = 6+h = 6.$

4. $\lim_{t \to 2} \frac{\frac{1}{t}-\frac{1}{2}}{t-2} = \frac{\frac{2-t}{2t}}{t-2} = -\frac{1}{2t} = -\frac{1}{4}.$

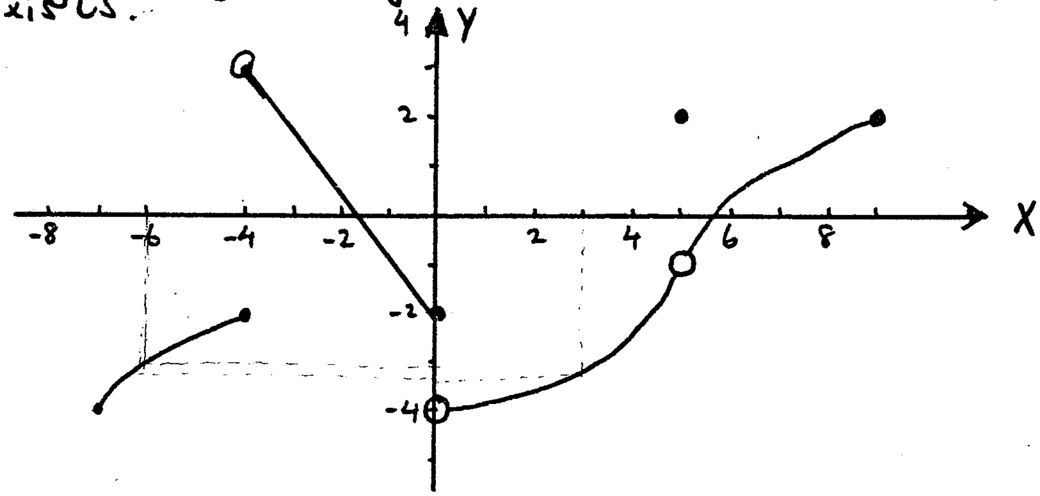
5. $\lim_{x \to 4} \frac{x^2-16}{\sqrt{x}-2} = \frac{\sqrt{x}+2}{\sqrt{x}+2} \quad \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}\sqrt{x}+2 = 8(4) = 32.$

Score: /10

One Sided Limits.

Name: Ans KEY

1. Use the given graph of f to state the value of the limit, if it exists.



a) $\lim_{x \to 0^+}$ -4 b) $\lim_{x \to 0^-}$ -2 c) $\lim_{x \to -4^+}$ 3

d) $\lim_{x \to 3}$ -3.5 (approx) e) $\lim_{x \to 0}$ no limit f) $\lim_{x \to -6}$ -3

2. Find the following limits, if they exist:

a) $\lim_{x \to 3^+} \sqrt{x-3}$ 0

b) $\lim_{x \to 6^+} |x-6|$ 0

c) $\lim_{x \to 0^-} \frac{|x|}{x}$ -1

d) $\lim_{x \to 0^+} \frac{|x|}{x}$ 1

e) $\lim_{x \to 0} \frac{|x|}{x}$ no limit

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Using limits to find Tangents　　　**Name:** Ans KF 7.

① Using the formula: $m = \lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$,

find the slope of the curve $y = x^2 - 6x + 5$ at $(2, -3)$. Show all work. (2)

$$m = \lim_{x \to 2} \frac{x^2 - 6x + 5 - (4 - 12 + 5)}{x - 2} = \lim_{x \to 2} \frac{x^2 - 6x + 8}{x - 2} = \lim_{x \to 2} \frac{(x-4)(x-2)}{x-2}$$

$$= \lim_{x \to 2} x - 4 = 2 - 4 = \boxed{-2}$$

② Using the formula: $m = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$, find the slope of the curve $y = \dfrac{1}{x-1}$ at $(3, \tfrac{1}{2})$.

(Show all work) (2)

$$m = \lim_{h \to 0} \frac{\frac{1}{3+h-1} - \frac{1}{3-1}}{h} = \lim_{h \to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h \to 0} \frac{2 - 2 - h}{2h(2+h)} = -\frac{1}{2(2+h)}$$

$$\boxed{= -\frac{1}{4}}$$

③ If the slope of the function $y = 4 - x^2$ at the point $(-2, 0)$ is 4, find the equation of the tangent at this point. (Show working) (2)

$$y - 0 = 4(x + 2)$$
$$= 4x + 8$$

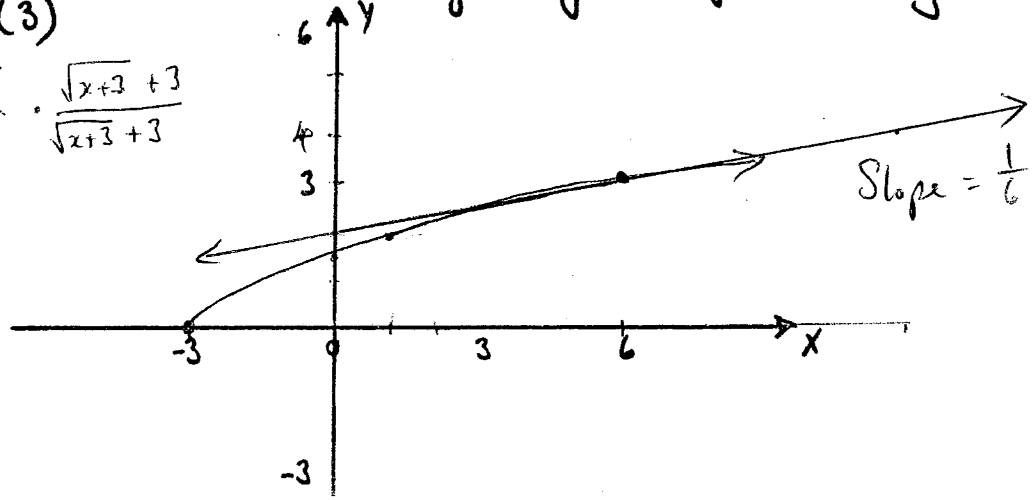
$$\boxed{4x - y + 8 = 0}$$

④ Sketch the curve and the tangent of the function $y = \sqrt{x+3}$, at $(6, 3)$. (3)

$$m = \lim_{x \to 6} \frac{\sqrt{x+3} - \sqrt{9}}{x - 6} \cdot \frac{\sqrt{x+3} + 3}{\sqrt{x+3} + 3}$$

$$= \lim_{x \to 6} \frac{\cancel{x+3-9}}{\cancel{x-6}[\sqrt{x+3} + 3]}$$

$$= \frac{1}{6}$$



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Using Limits to find Tangents Name: ______________________

① Find the slope of the tangent line to the curve $y = x^3 - 3x$ at the point $(1,-2)$, using the formula:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$m = \lim_{x \to a} \frac{x^3 - 3x - [1^3 - 3(1)]}{x - 1}$$

$$= \lim_{x \to a} \frac{x^3 - 3x + 2}{x - 1}$$

$$= \lim_{x \to a} x^2 + x - 2$$

1	1	0	-3	2
		1	1	-2
	1	1	-2	0

$$= 1^2 + 1 - 2$$

$$= \underline{0}.$$

(2)

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② Find the slope of the tangent line to the curve $y = x^2 + 5x + 6$ at the point $(-2, 0)$, using the formula:

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{(-2+h)^2 + 5(-2+h) + 6 - [(-2)^2 + 5(-2) + 6]}{h}$$

$$= \lim_{h \to 0} \frac{h^2 - 4h \cancel{+4} + 5h \cancel{-10} \cancel{+6} \cancel{-4} \cancel{+10} \cancel{-6}}{h}$$

$$= \lim_{h \to 0} h + 1 \quad = 1$$

(2)

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③ If the slope of a tangent line is -3 at a point $(5,2)$, find the equation of the tangent line in standard form.

$$y - 2 = -3(x - 5)$$

$$y - 2 = -3x + 15$$

$$3x + y - 17 = 0$$

(2)

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④ Graph the curve and the tangent of $y = 6 - \frac{1}{3}x^2$, at the point $(3,3)$

x	y
0	6
1	$5\frac{2}{3}$
2	$4\frac{2}{3}$
3	3
4	$\frac{2}{3}$

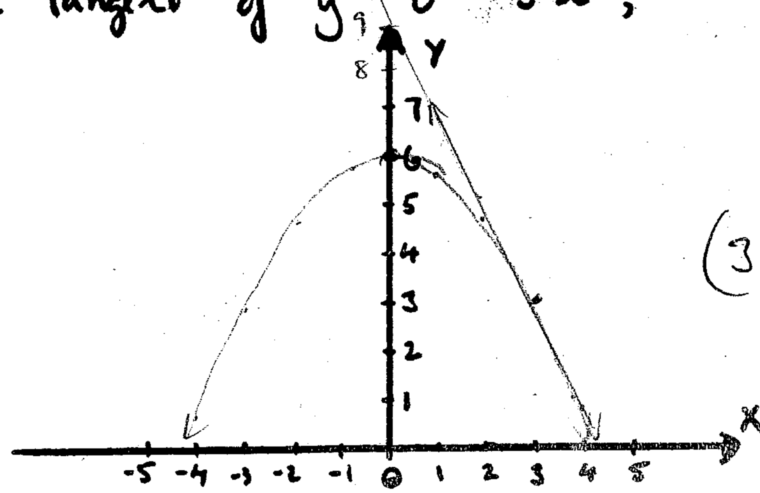
At $(3,3)$ $m = -\frac{2}{3}x$

$\underline{m} = -2$

$$y - 3 = -\cancel{2/3}(x - 3)$$

$$\cancel{3}y - \cancel{9} = -2x + 6$$

$$2x + \cancel{3}y = \cancel{15}\ 9$$



(3)

Infinite Sequences

Name: ANS KEY

① List the first six terms of the sequence defined by

$$t_n = \frac{n+2}{2n+1}$$

$\frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \frac{7}{11}, \frac{8}{13}$ (3)

$1, \frac{4}{5}, \frac{5}{7}, \frac{2}{3}, \frac{7}{11}, \frac{8}{13}$

② State the limit of each of the following sequences:

a) $2, 1\frac{1}{2}, 1\frac{1}{3}, 1\frac{1}{4} \ldots\ldots$ 1

b) $6, 5, 4, 5, 5\frac{1}{2}, 5, 4\frac{1}{2}, 5, 5\frac{1}{3}, 5, 4\frac{2}{3} \ldots$ 5

c) $3, 8, 3, 8, 3, 8, \ldots\ldots$ No limit

③ Find the following limits (if they exist). If there is no limit state 'no limit'.

a) $\lim\limits_{n\to\infty} \frac{3n}{2n+1}$ $\frac{3}{2}$

b) $\lim\limits_{n\to\infty} \frac{2n^2+5}{n^2-3}$ 2

c) $\lim\limits_{n\to\infty} \frac{2n^2+3n-5}{9-n^2}$ $\frac{2+3/n-5/n^2}{\frac{9}{n^2}-1} = -2$

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Infinite Series   Name: ______________________

1. Find the sum of each of the following series, or state that the series is divergent:

a) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots$ $a = 1$ $r = \frac{2}{3}$ $S = \frac{a}{1-r}$

$= \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3.$

b) $2 - \frac{3}{2} + \frac{9}{8} - \frac{27}{32} \ldots$ $a = 2$ $r = -\frac{3}{4}$ $S = \frac{a}{1-r}$

$= \frac{2}{1+\frac{3}{4}} = \frac{2}{\frac{7}{4}} = \frac{8}{7}$

c) $5 + 1 + 0.2 + 0.04 + \ldots$ $a = 5$ $r = 0.2$ $S = \frac{5}{1-0.2}$

$= \frac{5}{0.8} = \frac{50}{8} = \frac{25}{4}$ or $6\frac{1}{4}$

d) $\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$ $a = \frac{4}{5}$ $r = \frac{4}{5}.$ $S = \frac{\frac{4}{5}}{1-\frac{4}{5}} = \frac{\frac{4}{5}}{\frac{1}{5}} = 4.$

2. Express the following repeating decimals as an infinite series and hence find the rational equivalent:

a) $0.\overline{56}$ $= 0.56 + 0.0056$ $a = 0.56$ $r = 0.01$

$S = \frac{.56}{1-.01} = \frac{.56}{.99} = \frac{56}{99}$

b) $2.3\overline{4}$ $2.3\overline{4} = 2.3 + 0.04 + 0.004$ $S = \frac{0.04}{1-.1} = \frac{.04}{.9} = \frac{4}{90}$

$= \frac{23}{10} + \frac{4}{90} = \frac{207+4}{90} = \frac{211}{90}$

6.