1.7 Infinite Series

Sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Series: $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

$a = t_1 = \frac{1}{2}$ $\quad r = \frac{1}{2}$ $\quad S_\infty = \frac{a}{1-r}$

iff $|r|<1$ $\quad -1<r<1$

(if and only if)

If $|r|<1$ then S_∞ has a sum or limit & is said to be <u>CONVERGENT</u>

Otherwise, it is said to be <u>DIVERGENT</u>

Find the sum of:

① $\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^i = \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 \ldots$

Sigma "The Sum of..."

$a = \frac{1}{3}$ $\quad r = \frac{1}{3}$ ← between $-1<r<1$ so series is <u>CONVERG.</u>

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

② $\sum_{i=5}^{\infty} 8\left(\frac{1}{2}\right)^i = 8\left(\frac{1}{2}\right)^5 + 8\left(\frac{1}{2}\right)^6 + 8\left(\frac{1}{2}\right)^7 \ldots$

$\frac{1}{2}$ $\quad \frac{1}{2}$

t_1 or a

$a = \frac{8}{32} = \frac{1}{4}$ $\quad r = \frac{1}{2}$

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

③ $0.111111\overline{1}$

Gr 8ish: $x = 0.1111111\ldots$

$10x = 1.1111111\ldots$

$$\begin{aligned} 10x &= 1.111111\ldots \\ -\ x &= 0.111111\ldots \\ \hline 9x &= 1 \\ x &= \frac{1}{9} \end{aligned}$$

$$\begin{aligned} 100x &= 28.28282828\ldots \\ -\ x &= 0.28282828\ldots \\ \hline 99x &= 28 \\ x &= \frac{28}{99} \end{aligned}$$

<u>Using Series</u>: Write as a fraction

(1) $0.11111\ldots$

$0.1 + 0.01 + .001 + .0001 + \ldots$

$a = 0.1$ $\quad r = \frac{1}{10}$

$a = \frac{1}{10}$

$$S_\infty = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{9}$$

<u>Ex 2</u> $\quad 0.456\,123\,123\,123\ldots$

$0.456 + 0.000\,123\,123\ldots$

$\frac{456}{1000} +$ $\quad a = 0.000\,123$ $\quad t_2 = .000\,000\,123$

$r = \frac{1}{1000}$

$\frac{456}{1000}$

$$S_\infty = \frac{\frac{123}{1000000}}{1-\frac{1}{1000}}$$

$$S_\infty = \frac{\frac{123}{1000\,000}}{\frac{999}{1000}}$$

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$$S_\infty = \frac{123}{1000\cancel{000}} \cdot \frac{\cancel{1000}}{999}$$

$$= \frac{123}{1000} \cdot \frac{1}{999}$$

$$\frac{999}{999} \cdot \frac{456}{1000} + \frac{123}{1000 \cdot 999} = \frac{455667}{999000}$$

$$= \frac{151889}{333000}$$