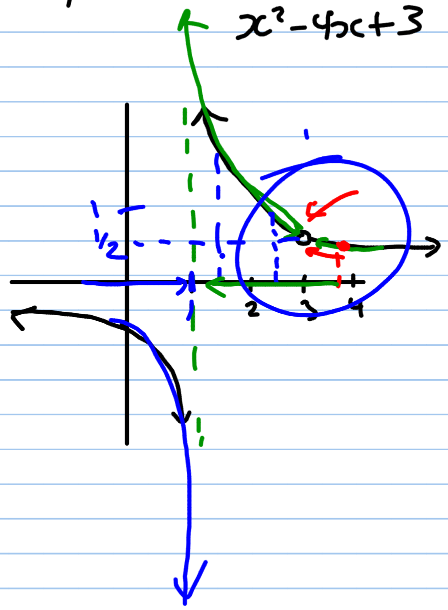
The Limit of a Function (1.2)

$$f(x) = \frac{x-3}{x^2-4x+3} = \frac{\cancel{(x-3)}}{\cancel{(x-3)}(x-1)} = \frac{1}{x-1}$$



what happens near $x=3$?

T.o.V.

$(x<3)$

x	$f(x)$
2.5	~~0~~ 2/3 .66667
2.9	0.526316
2.999	0.500250
2.9999	0.500025

$(x>3)$

x	$f(x)$
3.5	0.40
3.01	0.4975
3.0001	0.49997

English: The limit of $\frac{x-3}{x^2-4x+3}$ as x approaches 3 is $1/2$.

math: $\lim_{x\to3} \frac{x-3}{x^2-4x+3} = \frac{1}{2}$

In General:

$\lim_{x\to a} f(x) = L$ if we can make $f(x)$ arbitrarily close to L by taking values of x closer and closer to "a" (but not equal to "a")

NOTE The limit can exist even where a function does not.

Finding Limits

Situation 1: Direct substitution is permissible

$$\lim_{x \to 2} x^2+3x+1 = 2^2+3(2)+1 = \underline{\underline{11}}$$

If $\lim_{x \to a} f(x) = f(a)$ then the function is continuous at "a."

Situation 2: Direct substitution is <u>NOT</u> possible

<u>Ex 1</u> $\lim_{x \to 4} \frac{x^2-16}{x-4} = \lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}}$

$= \lim_{x \to 4} x+4 = 8$

$$\lim_{h\to 0}\frac{(2+h)^2-4}{h} = \lim_{h\to 0}\frac{\cancel{4}+\underline{\underline{4h+h^2}}\cancel{-4}}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{h}(4+h)}{\cancel{h}} = \underline{\underline{4}}$$

<u>Hints</u>:

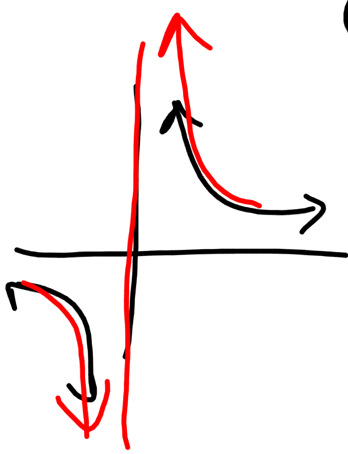
① FACTOR! (don't forget +/- cubes) Pg 2

② Rationalize denom/numerator

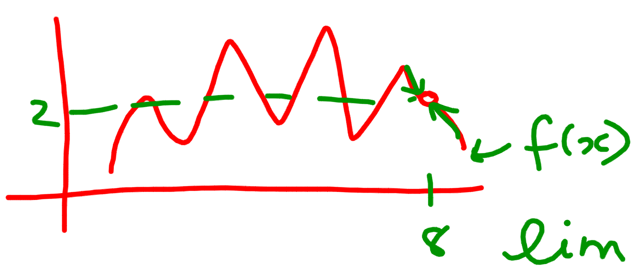
③ Deal with complex fractions

$$\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{5}} = \frac{\frac{3}{6}+\frac{2}{6}}{\frac{1}{5}} = \frac{\frac{5}{6}}{\frac{1}{5}} = \frac{5}{6}\cdot\frac{5}{1} = \frac{25}{6}$$

④ Check out the graph!



$$\lim_{x\to 0}\frac{1}{x} = \text{dne}$$ (does not exist)



$$\lim_{x\to 8} f(x) = 2$$