Infinite Sequences (1.6)

1, 2, 3, 4, 5, ...

$t_1 \rightarrow a$, t_4

2, 4, 8, 16, 32, ...

t_1 or a , t_4

1, 1, 2, 3, 5, 8, 13, 21, 34

Sequences with or without limits:

1, 2, 3, 4, 5, ... the limit dne

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$ limit would be 0

Ex: $t_n = \frac{n}{n+1}$

$$\lim_{n \to \infty} \frac{n}{n+1} = 1$$

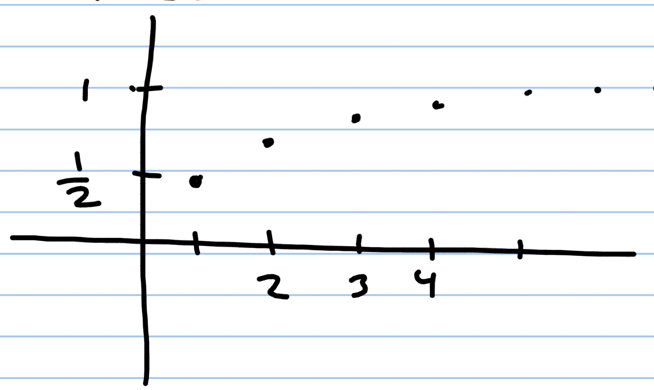
$t_1 = \frac{1}{1+1} = \frac{1}{2}$

$t_2 = \frac{2}{2+1} = \frac{2}{3}$

$t_3 = \frac{3}{3+1} = \frac{3}{4}$

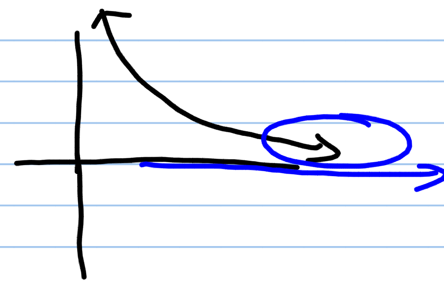
$t_4 = \frac{4}{5}$

$t_5 = \frac{5}{6}$



$t_{1000000} = \frac{1000000}{1000001}$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$



$$\lim_{n\to\infty} \frac{1}{n^2} = 0$$

In general: $\lim_{n\to\infty} \frac{1}{n^r} = 0$ if $r > 0$

Ex: Find $\underline{\underline{\lim_{n\to\infty}}} \frac{n^2-n}{2n^2+1}$

① Divide all terms by highest power of n

So: $$\lim_{n\to\infty} \frac{\cancel{\frac{n^2}{n^2}}^{\,1} - \cancel{\frac{n}{n^2}}^{\,\frac{1}{n}}}{\cancel{\frac{2n^2}{n^2}} + \frac{1}{n^2}}$$ REALLY $$\frac{\frac{1}{n^2}(n^2-n)}{\frac{1}{n^2}(2n^2+1)}$$

$$\lim_{n\to\infty} \frac{1 - \cancel{\frac{1}{n}}^{\,0}}{2 + \cancel{\frac{1}{n^2}}^{\,0}} = \frac{1-0}{2-0} = \frac{1}{2}$$

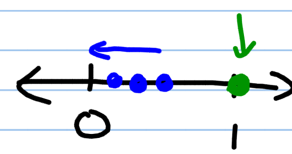
$$\lim_{n\to\infty} \frac{5n^5+3n^2-7}{3n^5+6n^4-3n^3+2} = \frac{5}{3}$$

$$\frac{6}{n} \qquad \frac{3}{n^3}$$

$$\lim_{n\to\infty} (-1)^n = \text{dne}$$ $-1, 1, -1, 1, -1, 1$

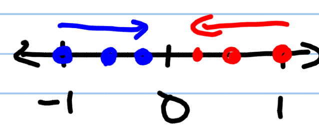
$$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4} \ldots$ limit dne

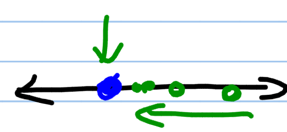


$-1, 1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, +\frac{1}{3} \ldots$

limit $\Rightarrow 0$



$0, 1, 0, \frac{1}{2}, 0, \frac{1}{4}, 0, \frac{1}{8} \ldots$



$$\lim_{n\to\infty} \frac{(n+1)^2}{n(n+2)} = \lim_{n\to\infty} \left(\frac{n^2+2n+1}{n^2+2n}\right) \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\lim_{n\to\infty} \frac{1+\frac{2}{n}+\frac{1}{n^2}}{1+\frac{2}{n}} = 1$$

Section 1.6