1.3 One-Sided Limits

A function, f^n, is a way of stating a rule.

$f(x) = x^2$ (The rule is that the value of the function is the x-value squared)

$$f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 3-x & \text{if } x > 1 \end{cases}$$

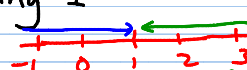
$f(-3) = 9$
$\longrightarrow f(1) = 1$
$f(7) = -4$

Let's look at $f(x)$ as it approaches the dividing line from either side.

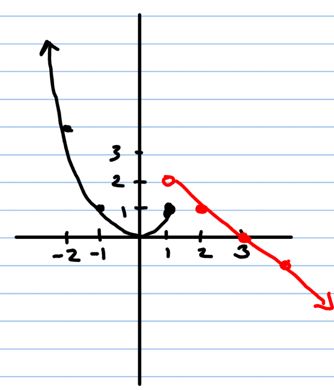
$x<1$	$f(x)=x^2$
0	0
.5	.25
.9	.81
.999	.998

$x>1$	$f(x)=3-x$
2	1
1.5	1.5
1.001	1.999
1.0001	1.9999

approaching 1 approaching 2



$\lim\limits_{x\to 1^-} f(x) = 1$ $\lim\limits_{x\to 1^+} f(x) = 2$



$f(x) = 3 - x$
2

If $\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$

then $\lim\limits_{x\to a} f(x)$ dne

If $\lim\limits_{x\to a^-} f(x) = L = \lim\limits_{x\to a^+} f(x)$

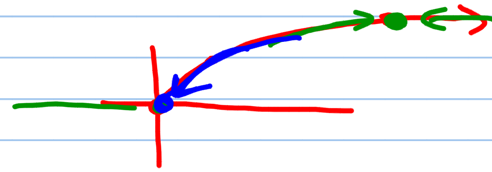
then $\lim\limits_{x\to a} f(x) = L$

Recall A graph is continuous at point a iff (if and only if)

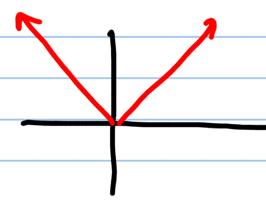
$$\lim_{x \to a} f(x) = f(a)$$

Ex

① $\lim_{x \to 0^+} \sqrt{x} = 0$

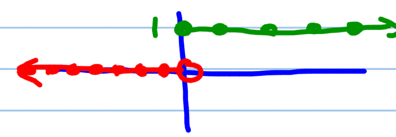


② $\lim_{x \to 0} |x| = 0$



$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

③ $H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$

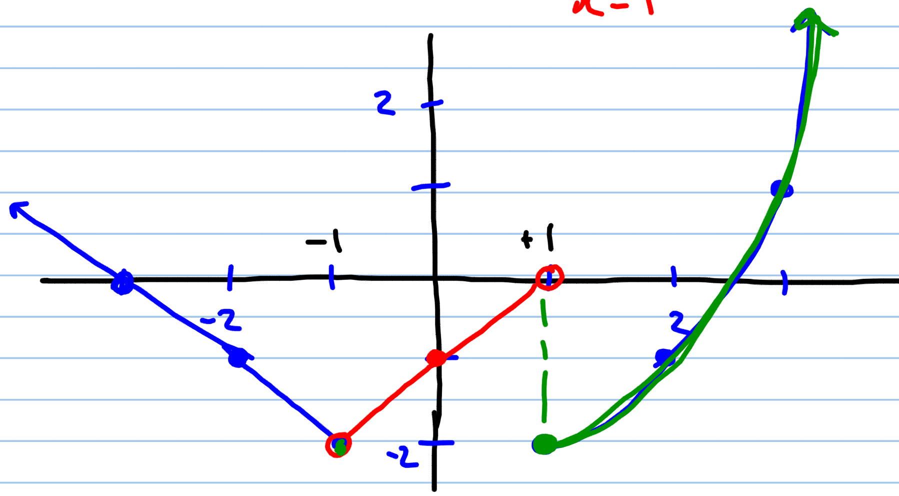


$\lim_{t \to 0^-} (H(t)) = 0$ $\lim_{t \to 0^+} H(t) = 1$

$$\lim_{t \to 0} H(t) = \text{dne}$$

$$f(x) = \begin{cases} -x-3 & \text{if } x \le -1 \\ x-1 & \text{if } -1 \underline{<} x < 1 \\ (x-1)^2-2 & \text{if } \underline{\underline{x \ge 1}} \end{cases}$$

$x=1^+$



$x=-1$

$$\lim_{x\to -1^-} -x-3 = -2$$

$$\lim_{x\to -1^+} x-1 = -2$$

$$\lim_{x\to -1} f(x) = -2$$

$x=1$

$$\lim_{x\to 1^-} x-1 = 0$$

$$\lim_{x\to 1^+} (x-1)^2-2 = -2$$

$$\lim_{x\to 1} f(x) = \text{dne}$$

1.3 ... enjoy!