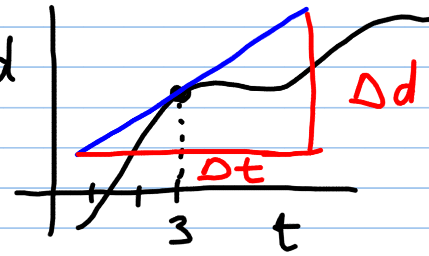
Velocity and other Rates of Change (1.5)

$v = \frac{\Delta d}{\Delta t}$



"Instantaneous" velocity

$v = \frac{\Delta d}{\Delta t}$ and we want the velocity at exactly 3.0s, then our $\Delta t \doteq 0$

We can solve this the same way as we did the tangent problem. which led us to:

① $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ OR ② $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Ex1 for a Falling object: $d = \cancel{v_0 t} + \frac{1}{2} a t^2$

$d = 4.9t^2$

What is v at exactly 3.0s?
"a"

$f(t) = 4.9t^2$

$v = \lim_{t \to 3} \frac{f(t) - f(3)}{t - 3} = \lim_{t \to 3} \frac{4.9t^2 - 4.9(3^2)}{t - 3}$

$\lim_{t \to 3} \frac{4.9(t^2 - 9)}{(t - 3)}$

$= \lim_{t \to 3} \frac{4.9\cancel{(t-3)}(t+3)}{\cancel{(t-3)}} = 4.9(3+3)$

$= 29.4 \text{ m/s}$

Ex2: The displacement (in m) of a particle moving in a straight line is given by:

$$s = t^2 + 2t$$

where t is time (s). Find the velocity after 3.0 s. $\Rightarrow a$

$$f(a+h) - f(a)$$

$$v(3) = m = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{(3+h)^2 + 2(3+h) - (3^2 + 2(3))}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 \cancel{+6} + 2h \cancel{-15}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 8h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+8)}{\cancel{h}} = \underline{\underline{8\ m/s}}$$

Other Rates of Change:

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \Rightarrow \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Applications

Physics → rate of change of disp. wrt time → velocity
→ Δv wrt Δt → acceleration

Chemistry → rate of change of [ ] wrt time

Manufacturing → marginal cost, marginal profit

→ can all be interpreted as problems involving tangent lines.