7.1 pg 306 #7-15 ODDS, 16, 20, 24, 28, 32, 36

7. $\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3 \sin 3x}{3x} = 3(1) = 3$

9. $\lim_{x \to 0} \frac{(2x)^3 \cdot \left(\frac{\sin 2x}{2x}\right)^3}{(3x)^3 \cdot \left(\frac{\sin 3x}{3x}\right)^3} = \frac{2^3}{3^3} = \frac{8}{27}$

10. $\lim_{x \to 0} \left(\frac{1-\cos x}{x}\right) \cdot \frac{(1+\cos x)}{(1+\cos x)} = \lim_{x \to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{\sin x}{1+\cos x}$

$= 1 \cdot \frac{0}{1+1} = 0$

11. $\lim_{x \to 0} (x^2 + \cos x) = \lim_{x \to 0} x^2 + \lim_{x \to 0} \cos x$

$= 0 + 1$

$= 1$

13. $\lim_{x \to \frac{\pi}{4}} \frac{\sin x}{3x} = \frac{\sin \pi/4}{3(\pi/4)} = \frac{\frac{1}{\sqrt{2}}}{\frac{3\pi}{4}}$

$= \frac{4\sqrt{2}}{3\pi \cdot 2} = \frac{2\sqrt{2}}{3\pi}$

15. $\lim_{x \to 0} \frac{\sin 5x}{5} = 0$

16. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\tan x}{4x} = \dfrac{\tan(\pi/4)}{\pi} = \dfrac{1}{\pi}$

20. $\lim\limits_{x \to 0} \dfrac{\sin 6x}{\cos 4x} = \dfrac{0}{1} = 0$

24. $\lim\limits_{x \to 0} \dfrac{(1 - \cos x)}{2x^2} \dfrac{(1+\cos x)}{(1+\cos x)}$

$= \lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{2x^2(1+\cos x)} = \lim\limits_{x \to 0} \dfrac{\sin^2 x}{2x^2(1+\cos x)}$

$= \lim\limits_{x \to 0} \dfrac{1}{2}\left(\dfrac{\sin x}{x}\right)^2 \cdot \lim\limits_{x \to 0} \dfrac{1}{1+\cos x}$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}$

28. $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2} = \lim\limits_{x \to 0} \dfrac{1 - (1 - 2\sin^2 x)}{x^2}$

$= \lim\limits_{x \to 0} 2\left(\dfrac{\sin x}{x}\right)^2 = 2(1) = \underline{\underline{2}}$

32. $\lim\limits_{x \to 0} \dfrac{\sin x}{\tan x} = \lim\limits_{x \to 0} \dfrac{\sin x}{\frac{\sin x}{\cos x}} = \lim\limits_{x \to 0} \cos x = 1$

36. $\lim_{x\to 0} \frac{\csc x - \cot x}{\sin x} = \lim_{x\to 0} \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{\sin x}$

$= \lim_{x\to 0} \frac{\frac{1-\cos x}{\sin x}}{\sin x}$

$= \lim_{x\to 0} \frac{1-\cos x}{\sin^2 x}$

$= \lim_{x\to 0} \frac{1-\cos x}{1-\cos^2 x}$

$= \lim_{x\to 0} \frac{(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$= \lim_{x\to 0} \frac{1}{1+\cos x} = \frac{1}{1+1} = \frac{1}{2}$

7.2 Pg 313 #1 ace's, 2ac, 3ac.

1. a) $y = \cos(-4x)$

$y' = 4\sin(-4x)$ or $y' = -4\sin(4x)$

c) $y = 4\sin(-2x^2-3)$

$y' = 4\cos(-2x^2-3)(-4x)$

$= -16x\cos(-2x^2-3)$

e) $y = \sin(x^2)$

$y' = 2x\cos x^2$

g) $y = \sin^{-2}(x^3)$ $\quad (\sin(x^3))^{-2}$

$-2(\sin(x^3))^{-3} \cos(x^3)\ 3x^2$

$y' = -2(\sin^{-3}(x^3)) \cdot \cos(x^3) \cdot 3x^2$

$= \dfrac{-6x\cos(x^3)}{\sin^3(x^3)}$

i) $y = 3\sin^4(2-x)^{-1}$ $\quad 3(\sin(2-x)^{-1})^4$

$y' = 12\sin^3(2-x)^{-1} \cdot \cos(2-x)^{-1} \cdot -1(2-x)^{-2} \cdot (-1)$

$= \dfrac{12\sin^3(2-x)^{-1} \cdot \cos(2-x)^{-1}}{(2-x)^2}$

k) $y = \dfrac{x}{\sin x}$

$y' = \dfrac{\sin x(1) - x\cos x}{\sin^2 x}$

$= \dfrac{\sin x - x\cos x}{\sin^2 x}$

m) $y = (1+\cos^2 x)^6$

$y' = 6(1+\cos^2 x)^5 \cdot 2\cos x \cdot (-\sin x) \cdot (1)$

$= -12(1+\cos^2 x)^5 \cos x \sin x$

or $= -6(1+\cos^2 x)^5 \cdot \underbrace{2\cos x \sin x}$

$= -6(1+\cos^2 x)^5 \sin 2x$

o) $y = \sin(\cos x)$

$\sin(m)$

$\cos(m) \cdot m'$

$y' = \cos(\cos x) \cdot -\sin x$

$= -\sin x \cos(\cos x)$

q) $y = x\left(\cos \frac{1}{x}\right)$

$y' = x \cdot -\sin\left(\frac{1}{x}\right) \cdot (-x^{-2}) + \cos\frac{1}{x} \cdot 1$

$= \frac{\sin \frac{1}{x}}{x} + \cos\frac{1}{x}$

or $\frac{\sin\frac{1}{x} + x\cos\frac{1}{x}}{x}$

s) $y = \frac{1+\sin x}{1-\sin 2x}$ $-\cos 2x$

$y' = \frac{(1-\sin 2x)(\cos x) - (1+\sin x)(-2\cos 2x)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x \sin 2x + 2\cos 2x + 2\sin x \cos 2x}{(1-\sin 2x)^2}$

u) $y = \cos^2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)$

$$y' = 2\cos\left[\frac{1-\sqrt{x}}{1+\sqrt{x}}\right] \cdot -\sin\left[\frac{1-\sqrt{x}}{1+\sqrt{x}}\right] \cdot \left[\frac{(1+\sqrt{x})(-\frac{1}{2}x^{-1/2}) - \overbrace{(\tfrac{1}{2}x^{-1/2})}^{(1-\sqrt{x})}}{(1+\sqrt{x})^2}\right]$$

$$= -\sin 2\left[\frac{1-\sqrt{x}}{1+\sqrt{x}}\right]\left[\frac{-1/2\,x^{-1/2}(1+\sqrt{x}+1-\sqrt{x})}{(1+\sqrt{x})^2}\right]$$

$$= \sin 2\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left[\frac{1}{x^{1/2}(1+\sqrt{x})^2}\right]$$

$$= \sin 2\left[\frac{1-\sqrt{x}}{1+\sqrt{x}}\right]\left[\frac{1}{\sqrt{x}(1+\sqrt{x})^2}\right]$$

2. a) $\sin y = \cos 2x$.

$$\frac{d}{dx}\sin y = \frac{d}{dx}\cos 2x$$

$$\cos y \cdot \frac{dy}{dx} = -\sin 2x\,(2)$$

$$\frac{dy}{dx} = \frac{-2\sin 2x}{\cos y}$$

c) $\sin y + y = \cos x + x$

$$\frac{d}{dx}\sin y + \frac{d}{dx}y = \frac{d}{dx}\cos x + \frac{d}{dx}x$$

$$\cos y\frac{dy}{dx} + \frac{dy}{dx} = -\sin x + 1$$

$$\frac{dy}{dx}(\cos y + 1) = -\sin x + 1$$

$$\frac{dy}{dx} = \frac{1-\sin x}{\cos y + 1}$$

3. a) $y = 2\sin x$ at $\left(\frac{\pi}{6}, 1\right)$

$$y' = 2\cos x$$

$$y'\left(\frac{\pi}{6}\right) = 2\cos\frac{\pi}{6}$$

$$= 2\frac{\sqrt{3}}{2} = \sqrt{3}$$

$$y - 1 = \sqrt{3}(x - \pi/6)$$

$$y - 1 = \sqrt{3}x - \frac{\sqrt{3}\pi}{6}$$

$$6y - 6 = 6\sqrt{3}x - \sqrt{3}\pi$$

$$6\sqrt{3}x - 6y - \pi\sqrt{3} + 6 = 0$$

c) $y = \frac{1}{\cos x} - 2\cos x$ at $\left(\frac{\pi}{3}, 1\right)$

$$y' = -(\cos^{-2}x)(-\sin x) - 2(-\sin x)$$

$$= +\frac{\sin x}{\cos^2 x} + 2\sin x$$

$$y'\left(\frac{\pi}{3}\right) = +\frac{\sin \pi/3}{\cos^2(\pi/3)} + 2\sin \pi/3$$

$$= \frac{+\sqrt{3}/2}{(1/2)^2} + 2\,\cancel{\sin}\,\sqrt{3}/2$$

$$= +2\sqrt{3} + \frac{2\sqrt{3}}{2} = 3\sqrt{3}$$

$y-1=3\sqrt{3}(x-\frac{\pi}{3})$

$y-1=3\sqrt{3}x-\sqrt{3}\pi$

$3\sqrt{3}x-y+1-\pi\sqrt{3}=0$

7.3 pg 319 #1aces, 2a, 3a

1. a) $y=3\tan 2x$

$y'=6\sec^2 2x$

c) $y=12\sec \frac{1}{4}x$

$y'=3\sec\frac{1}{4}x\tan\frac{1}{4}x$

e) $y=\tan x^2$

$y'=2x\sec^2 x^2$

g) $y=\sec(x)^{1/3}$

$y'=\frac{1}{3}x^{-2/3}(\sec x^{1/3}\tan x^{1/3})$

i) $y=\cot^3(1-2x)^2$

$y'=3\cot^2(1-2x)^2\cdot -\csc^2(1-2x)^2\cdot 2(1-2x)\cdot -2$

$=12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

k) $y = (\sec(2x) - 1)^{-3/2}$

$y' = \frac{-3}{\cancel{2}}(\sec 2x - 1)^{-5/2} \cdot \cancel{2} \sec 2x \tan 2x$

$= \frac{-3 \sec 2x \tan 2x}{(\sec 2x - 1)^{5/2}}$

m) $y = 2x(\sqrt{x} - \cot x)$

$y' = 2x(\frac{1}{2}x^{-1/2} + \csc^2 x) + 2(\sqrt{x} - \cot x)$

$= \sqrt{x} + 2x\csc^2 x + 2\sqrt{x} - 2\cot x$

$= 3\sqrt{x} + 2x\csc^2 x - 2\cot x$

o) $y = \tan^2(\cos x)$

$y' = 2\tan(\cos x) \cdot \sec^2(\cos x) \cdot -\sin x$

$= -2\sin x \tan(\cos x)\sec^2(\cos x)$

2. a) $\tan x + \sec y - y = 0$

$\frac{d}{dx}\tan x + \frac{d}{dx}\sec y - \frac{d}{dx}y = \frac{d}{dx}0$

$\sec^2 x + \sec y \tan y \frac{dy}{dx} - \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-\sec^2 x}{\sec y \tan y - 1}$

3. a) $y = \cot^2 x$ when $x = \frac{\pi}{4}$

$y = \cot^2\left(\frac{\pi}{4}\right) = 1$ so pt is $\left(\frac{\pi}{4}, 1\right)$

$y' = \cancel{t \csc^2 x}$

$y' = -2\cot x \cdot \csc^2 x$

$y'\left(\frac{\pi}{4}\right) = -2\cot\frac{\pi}{4} \cdot \csc^2\frac{\pi}{4}$

$= -2 \cdot 1 \cdot (\sqrt{2})^2 = -4$

$y - 1 = -4\left(x - \frac{\pi}{4}\right) \rightarrow y - 1 = -4x + \pi$

$4y \cancel{- 4 - 16x - 4\pi}$ $4x + y - \pi - 1 = 0$

duh.

7.6 pg 339

1. a) $y = \sin^{-1}(x+1)$

$y' = \cancel{\frac{1}{\sqrt{1-(x+1)^2}}} \quad \frac{1}{\sqrt{1-(x+1)^2}} = \frac{1}{\sqrt{1-x^2-2x-1}} = \frac{1}{\sqrt{-x^2-2x}}$

c) $y = \tan^{-1}(3x)$

$y' = \frac{3}{1+9x^2}$

e) $y = \cos^{-1}\left(\frac{x^3}{2}\right)$

$y' = \frac{-1}{\sqrt{1-\left(\frac{x^3}{2}\right)^2}} \cdot \frac{3x^2}{2} \rightarrow \frac{-3x^2}{2\sqrt{1-\frac{x^6}{4}}} = \frac{-3x^2}{\sqrt{4\cdot\left(1-\frac{x^6}{4}\right)}}$

$= \frac{-3x^2}{\sqrt{4-x^6}}$

7.3

g. $y = \cos^{-1}\sqrt{2x-1}$

$$y' = \frac{-1}{\sqrt{1-(\sqrt{2x-1})^2}} \cdot \frac{1}{\cancel{2}}(2x-1)^{-\frac{1}{2}} \cdot \cancel{2}$$

$1-(2x-1)$

$$= \frac{-1}{(2x-1)^{\frac{1}{2}} \cdot \sqrt{1-2x+1}} = \frac{-1}{(2x-1)^{\frac{1}{2}}(2-2x)^{\frac{1}{2}}}$$

i) $y = \sin^{-1}\left[\frac{\cos x}{1+\sin x}\right]$

$$y' = \frac{1}{\sqrt{1-\left[\frac{\cos x}{1+\sin x}\right]^2}} \cdot \left[\frac{(1+\sin x)(-\sin x)-(\cos x \cdot \cos}{(1+\sin x)^2}\right.$$

$$y' = \frac{1}{\text{mess}} \cdot \left[\frac{-\sin x-(\sin^2 x + \cos^2 x)}{(1+\sin x)^2}\right]$$

$(1+\sin x)$

$$= \frac{1}{\text{mess}} \cdot \frac{-(\cancel{\sin x + 1})}{(1+\sin x)^2}$$

$$= \frac{-1}{\sqrt{\left[1-\frac{(\cos x)^2}{(1+\sin x)^2}\right] \cdot (1+\sin x)^2}}$$

$$= \frac{-1}{\sqrt{(1+\sin x)^2-\cos x^2}}$$

7.3

$$= \frac{-1}{\sqrt{1 + 2\sin x + \sin^2 x - \cos^2 x}}$$

$$= \sin^2 x$$

$$= \frac{-1}{\sqrt{\sin^2 x + 2\sin x + \sin^2 x}}$$

$$= \frac{-1}{\sqrt{2\sin x + 2\sin^2 x}}$$

$$= \frac{-1}{\sqrt{2\sin x\,(1 + \sin x)}}$$

Bleah!

5. $y = \tan x$

$y' = \sec^2 x$

$y'\left(\frac{\pi}{3}\right) = \sec^2\left(\frac{\pi}{3}\right) = \frac{1}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{1}{2}\right)^2}$

so $m = 4$

PT: $y = \tan x$

$y = \tan\left(\frac{\pi}{3}\right)$ $\quad x = \frac{\pi}{3}$

$y = \sqrt{3}$ so $\left(\frac{\pi}{3}, \sqrt{3}\right)$

$y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)$

$y = 4x - \frac{4\pi}{3} + \sqrt{3}$

$12x - 3y - 4\pi + 3\sqrt{3} = 0$

$$y = \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}} \cdot \left(\frac{(x+1)(1)-(1)(x-1)}{(x+1)^2}\right)$$

$$= \frac{1}{\sqrt{\frac{(x+1)^2-(x-1)^2}{(x+1)^2}}} \cdot \frac{x+1-x+1}{(x+1)^2}$$

$$= \frac{1}{\frac{\sqrt{(x+1)^2-(x-1)^2}}{(x+1)}} \cdot \frac{2}{(x+1)^2}$$

$$= \frac{2}{\sqrt{x^2+2x+1-(x^2-2x+1)}} \cdot \frac{1}{(x+1)}$$

$$= \frac{2}{(x+1)\sqrt{4x}} = \frac{\cancel{2}}{(x+1)\cancel{2}\sqrt{x}}$$

$$= \frac{1}{(x+1)\sqrt{x}}$$