3.1 Pg 124

1. a) $V_0 = 0$
   b) C → slope of tangent is steeper
   c) A - speeding up
      B - Slowing Down
      C - Speeding up
   d) $v = 0$ (it is stopped)
   e) The car returned to its starting point.

2. a) $s' = 12$ $\quad s'(2) = s'(4) = 12$ m/s.

   b) $s' = 16t - 24$
   $s'(2) = 16(2) - 24 = 8$ m/s
   $s'(4) = 16(4) - 24 = 40$ m/s.

   c) $s' = 3t^2 - 12t$
   $s'(2) = 3 \cdot 4 - 12 \cdot 2$
   $= -12$ m/s
   $s'(4) = 3(16) - 12(4)$
   $= 48 - 48 = 0$ m/s.

   d) $s' = \frac{(1+t)(5) - (1)(5t)}{(1+t)^2} = \frac{5}{(1+t)^2}$

   $s'(2) = \frac{5}{3^2} = 5/9$ m/s $\quad s'(4) = \frac{5}{25} = \frac{1}{5}$ m/s

3. $h' = -15 - 9.8t$

$h'(1) = -24.8\ m/s$

$h'(2) = -15 - 19.6$

$= -34.6\ m/s$

4. $h' = 24.5 - 9.8t$

a) $h'(1) = 14.7\ m/s$

$h'(2) = 4.9\ m/s$

$h'(3) = -4.9\ m/s$

$h'(4) = -14.7\ m/s.$

b) max height is when $v = 0$

so: $24.5 - 9.8t = 0$

$24.5 = 9.8t$

$t = 2.5s$

c) $h = 24.5(2.5) - 4.9(2.5^2)$

$h = 30.6m$

d) after 5.0s

e) $-24.5\ m/s.$

5. $s' = 320t + 20$

$100 = 320t + 20$

$-20 \qquad -20$

$\frac{80}{320} = \frac{320t}{320}$

$t = 1/4\ hr.$

8. $s' = 3t^2 - 30t + 63$

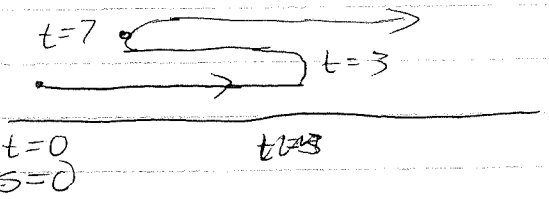
a) $0 = 3t^2 - 30t + 63$
$0 = 3(t^2 - 10t + 21)$
$0 = 3(t-7)(t-3)$
$t = 3, 7$
It is at rest at $t = 3$ & 7 seconds

(b) $3t^2 - 30t + 63 > 0$
$t^2 - 10t + 21 > 0$
$(t-7)(t-3) > 0$

Test ① $(2)^2 - 10(2) + 21 > 0$
$4 - 20 + 21 > 0$ ✓
② $(4)^2 - 10(4) + 21 > 0$
$16 - 40 + 21$
$-3 \not> 0$
③ $(8^2) - 80 + 21$
$64 - 80 + 21$
$5 > 0$

so $0 \le t < 3$ and $t > 7$

c)



d) ~~$|s(3) - s(0)| = |3 \cdot 9 - 30 \cdot 3 +$~~
oops

d) $s = t^3 - 15t^2 + 63t$

$s(3) = (3^3) - 15(9) + 63(3)$
$= 81\text{m}$

$s(7) = 7^3 - 15(49) + 63(7)$
$= 49$

so $|s(7) - s(3)| = 32\text{m}$

$s(10) = 1000 - 1500 + 630$
$= 130\text{m}$

so $|s(10) - s(7)| = 81$

Total distance: 194m

9. $s' = 10 - 10t$

a) $0 = 10 - 10t$
$t = 1.0\text{s}$

b) $s = 0$ so: $0 = -5t^2 + 10t + 450$
$= -t^2 + 2t + 90$

$$t = \frac{-2 \pm \sqrt{4 - 4(-1)(90)}}{-2}$$

$$= \frac{-2 \pm \sqrt{364}}{-2}$$

$$= \frac{-2 \pm 2\sqrt{91}}{-2} \quad = +1 \pm \sqrt{91} \quad \text{(reject -ve answers)}$$

$\approx 1 + \sqrt{91} \approx 10.5\text{s}$

c) $s'(10.5) = 10 - 10(10.5)$
$= -95 \text{ m/s}$

3.2 ACCELERATION

1. a) O – A : positive
b) A – B : negative
c) B – C : positive
d) C – D : zero
e) D – E : positive

2. a) O → A
– the velocity is increasing
∴ acceleration is positive
b) (i) A → B : – negative
(ii) B → C : – positive
(iii) C → D : – zero
(iv) D → E : – negative

3. a) $s' = 30 = v$
$s'' = 0 = a$ 3.2

b) velocity $= s' = 32t + 5$
accel^{n} $= s'' = 32$
oops.

c) vel: $s' = 15 - 9.8t$ $v = 3t^2 + 10t + 1$
$a = s'' = -9.8$ $a = 6t + 10$

d) $s' = \frac{1}{2}(t^2 + t)^{-1/2}(2t + 1) = \frac{2t+1}{2\sqrt{t^2+t}}$

$$s'' = \frac{2(t^2+t)^{1/2}(2) - (t^2+t)^{-1/2}(2t+1)}{4(t^2+t)}$$

$s' = \frac{2t+1}{2\sqrt{t^2+t}}$ · $(2t+1)$

$$= \frac{4(t^2+t)^{1/2}\cdot(t^2+t)^{1/2} - (2t+1)(2t+1)}{(t^2+t)^{1/2}} \Big/ 4(t^2+t)$$

$$= \frac{4t^2+4t-(4t^2+4t+1)}{4(t^2+t)^{3/2}} = \frac{-1}{4(t^2+t)^{3/2}}.$$

$= \frac{4t^2+2t+1}{4(t^2+t)^{3/2}}$ oops again

4. a) $s' = -15\$ - 9.8t$

$s'' = -9.8$

$s''(4) = -9.8\ m/s^2$

b) $s' = 3t^2 - 2t$ 3.2

$s'' = 6t - 2$

$s''(4) = 24 - 2 = 22\ m/s^2$

5. $s(0) = s_0$

$s'(0) = v_0 + gt$

$= v_0 + g(0) = v_0$

$s'' = g$

$s''(0) = g.$

Hilary

6. $s' = 3t^2 - 12$

$3t^2 - 12 = 0$

$3t^2 = 12$

$t^2 = 4$

$t = 2$ seconds.

3.2

$a = 6t$

$a(2) = 12 \text{ m/s}^2$

Exercise 3.3 Pg 133-134

1. $V = x^3$

$\frac{dV}{dx} = 3x^2$

$V'(4) = 3(16)$

$= 48 \text{ units}^3$

2. $A = \pi r^2$

$A' = 2\pi r$

$A'(5) = 10\pi \text{ cm}^2/\text{cm}$

3. $V' = 2\left(1000\left[1 - \frac{t}{60}\right]'\right)\left(\frac{-1}{60}\right)$

$= -\frac{2000}{60}\left[1 - \frac{t}{60}\right]$

$= -\frac{100}{3}\left[1 - \frac{t}{60}\right]$

$V'(10) = -\frac{100}{3}\left[1 - \frac{10}{60}\right] \approx -27.78 \text{ L/min}$

or $\frac{250}{9}$ L/min

4. a) $m = \sqrt{x}$

$\frac{\Delta m}{\Delta x} = \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{\sqrt{1.1} - \sqrt{1}}{.1} = 0.488 \text{ kg/m}$

(b) $m' = \frac{1}{2}x^{-1/2}$

$m'(1) = \frac{1}{2} \text{ kg/m}$

3.3

5. $m' = 1 + x$

$m'(6) = 1 + 6$

$= 7$ g/cm

6. $n' = 180 + 50t + 9t^2$

$n'(3) = 180 + 50(3) + 9(9)$

$= 411$ units/hr

7. $\beta = -\frac{1}{V}\frac{dV}{dP}$ $\qquad \frac{dV}{dP} = \frac{-5.3}{P^2}$ $\qquad V = \frac{5.3}{P} = 5.3P^{-1}$

$$\beta = \frac{-1}{\frac{5.3}{P}} \cdot \frac{-5.3}{P^2} = \frac{1}{P} = \frac{1}{40}$$

Exercise 3.4 pg 139

1. (a) $C'(x) = 23 + 0.024x$

(b) $C'(100) = 23 + 0.024(100)$

$= \$25.40$/~~100~~ items

(c) $C(101) - C(100) =$

$55000 + 23(101) + 0.012(101)^2 -$

$(55000 + 23(100) + .012(100)^2)$

$= \$25.41$

2. (a) $C'(x) = \frac{1}{10} + \frac{x}{500}$

(b) $C'(800) = \frac{1}{10} + \frac{800}{500} = \$1.70$/unit

(c) $C(801) - C(800) = 1500 + \frac{801}{10} + \frac{801^2}{1000} - \left(1500 + \frac{800}{10} + \frac{800^2}{1000}\right)$

$= 721.701 - 720$

$= \$1.701$

3. (a) $R'(x) = 8000 - .06x^2$

(b) $R'(300) = 8000 - .06(300^2)$

$= \$2600/\text{unit}$

(c) $R(301) - R(300)$

$= [8000(301) - 0.02(301)^3] - [8000(300) - .02(30$

$= 1862581.98 - 1860000$

$= \$2581.98$

4. (a) $P(x) = R(x) - C(x)$

$P(x) = 0.98x - .0002x^2 - 23000 - 0.24x - .0001x$

$P(x) = 0.74x - .0003x^2 - 23000$

(b) $P'(x) = 0.74 - 0.0006x$

(c) $P'(1000) = 0.74 - 0.0006(1000)$

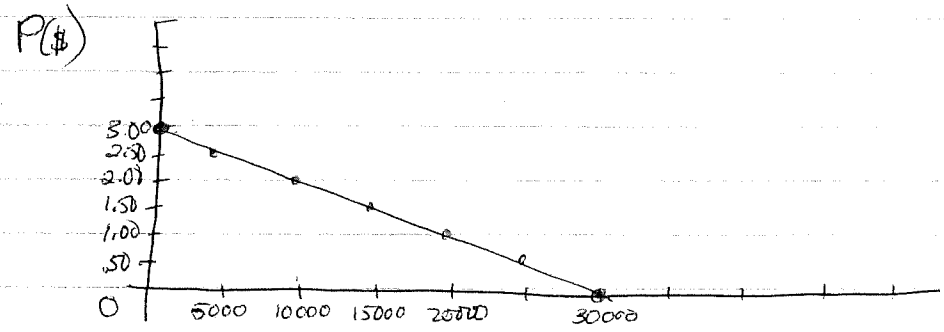
$= \$0.14/\text{Pen}$

(d) omit

5. $p = \frac{30000 - x}{10000}$ $\quad C(x) = 6000 + 0.8x$

(a+b)

P	0	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00
x	30000	25000	20000	15000	10000	5000	0



(c) $R(x) = x\,(p(x))$

$= x\left[\frac{30000 - x}{10000}\right] = \frac{1}{10000}\left[30000x - x^2\right]$

Hilbert

d) $R'(x) = \frac{1}{10000}[30000 - 2x]$

e) $R'(1000) = \frac{1}{10000}[30000 - 2(1000)]$

$= \frac{1}{10000}[28000]$

$= \$2.80/\text{unit}^{(sub)}$

f) $P(x) = R(x) - C(x)$

$= \frac{1}{\$0000}[30000x - x^2] - [6000 + .8x]$

$= 2.2x - .0001x^2 - 6000$

g) $P'(x) = 2.2 - 0.0002x$

h) $P'(10000) = 2.2 - 0.0002(10000)$

$= \$0.20/\text{sub}$

3.5 pg 145

1. $xy^2 = 12$ $\quad \frac{dy}{dt} = 6$ Find $\frac{dx}{dt}$ when $y = 2$

$$\frac{d}{dt}(xy^2) = \frac{d}{dt}(12)$$

$$\frac{d}{dt}(xy^2) = 0$$

$$x \cdot 2y\left[\frac{dy}{dt}\right] + \frac{dx}{dt} \cdot y^2 = 0$$

$$\frac{\frac{dx}{dt} y^2}{y^2} = \frac{-x \cdot 2y\,[dy/dt]}{y^2}$$

$$= \frac{dx}{dt} = \frac{-x\,2y\,[dy/dt]}{y}$$

From $xy^2 = 12$

$$x = \frac{12}{y^2}$$

$$\frac{dx}{dt} = \frac{-[12/y^2][2][dy/dt]}{y}$$

Sub in $y = 2$ & $\frac{dy}{dt} = 6$

$$\frac{dx}{dt} = \frac{-[12/4][2][6]}{2}$$

$$= -18$$

2. $\frac{d}{dt}[x^3 + y^3] = \frac{d}{dt}[9]$

$$3x^2 \cdot \frac{dx}{dt} + 3y^2 \frac{dy}{dt} = 0$$

$$3y^2 \frac{dy}{dt} = -3x^2 [dx/dt]$$

$$\frac{dy}{dt} = \frac{-3x^2 [dx/dt]}{3y^2} \qquad y = \sqrt[3]{9 - x^3}$$

$$= \frac{-x^2 [dx/dt]}{(\sqrt[3]{9-x^3})^2} \qquad \text{sub in } x = 2, \ \frac{dx}{dt} = 4$$

$$\frac{dy}{dt} = \frac{-[2^2][4]}{(\sqrt[3]{9-8})^2}$$

$$= -16$$

3. $A = s^2$ want $\frac{dA}{dt}$ $\frac{ds}{dt} = +0.8 \text{ m/min}$

$$\frac{dA}{dt} = 2s \cdot \frac{ds}{dt} \qquad s = 3, \ \frac{ds}{dt} = .8$$

$$\frac{dA}{dt} = 2(3)(.8) = 4.8 \text{ m}^2/\text{min}$$

4. $V = s^3$ want $\frac{ds}{dt}$ $\frac{dV}{dt} = 144\ cm^3/s$

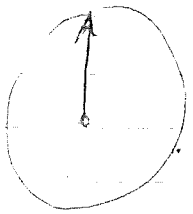
$s = 4\ cm$

$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$144 = 3(16) \cdot \frac{ds}{dt}$

$\frac{ds}{dt} = \frac{144}{48}$

$\frac{ds}{dt} = 3\ cm/s$



5. $A = \pi r^2$ Want $\frac{dA}{dt}$ $\frac{dr}{dt} = 25\ cm/s$

at $t = 4.0\ s$.

$r = 100\ cm$ (25×4)

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

$= 2\pi(100)(25)$

$= 5000\pi\ cm^2/s \approx 15707.96\ cm^2/s$

6. $V = \frac{4\pi r^3}{3}$ $\frac{dV}{dt} = 8\ m^3/min$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $r = \frac{d}{2} = 1\ m$

Find $\frac{dr}{dt}$

$8 = 4\pi(1)^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{8}{4\pi} = \frac{2}{\pi} \approx 0.637\ m/min$

7. $A = 4\pi r^2$ $\quad \frac{dA}{dt} = 0.5 \text{ cm}^2/\text{min}$

Find $\frac{dr}{dt}$ when $r = 4$

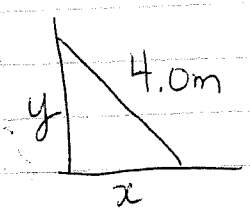
$$\frac{dA}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{dA/dt}{8\pi r}$$

$$= \frac{0.5}{8\pi(4)}$$

$$\approx 0.005 \text{ cm/min}$$

★ 11.



$x^2 + y^2 = 16$ $\quad \frac{dx}{dt} = 30$ cm/s when $x = 2$.

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(16)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$y = \sqrt{-x^2+16}$

so. $\frac{dy}{dt} = \frac{-2x\, dx/dt}{2y}$

$$= \frac{-x\, dx/dt}{\sqrt{-x^2+16}} = \frac{-2(30)}{\sqrt{12}} = \frac{-60}{2\sqrt{3}}$$

$$= \frac{-30\sqrt{3}}{3}$$

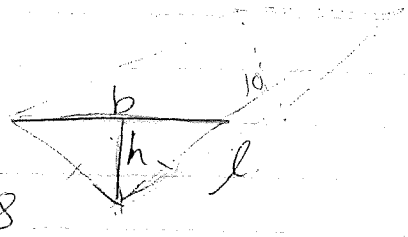
$$= \frac{-\sqrt{3}}{10} \approx 0.17 \text{ m/s}$$

16) $V = \frac{1}{2}bh \cdot \ell$

$V = \frac{1}{2}(2h)(h) \cdot \ell$ ← since $\ell = 10m$ always

$V = h^2\ell = 10h^2$

$\frac{dV}{dt} = 20h \frac{dh}{dt}$



$b = 2h$

$b = 1\ m$
$h = .5\ m$

$0.4 = 20h \frac{dh}{dt}$

want $\frac{dh}{dt}$ at ~~h~~=

$\frac{dV}{dt} = 0.4\ m^3/mi$

$\frac{dh}{dt} = \frac{0.4}{20h}$

$\frac{dh}{dt} \left(\frac{0.4}{20(.40)}\right)$??

$\frac{dh}{dt}(.40) = \frac{0.4}{20(.40)} = 0.05\ m/min$