5.2 Multiplying and Dividing Radicals

When multiplying radicals, there are two important things to note:

① The INDEX on the radicals MUST be the same or you can do Nothing

ex: $\sqrt[3]{2} \cdot \sqrt[3]{3} = \sqrt[3]{6}$ (indexes are both 3 so go ahead and multiply inside)

$\sqrt[5]{x} \cdot \sqrt[6]{3}$ ... Can't do anything as the indexes don't match.

Remember:

index $\rightarrow$ $\sqrt[5]{x}$ } the whole thing is the radical

radicand

② Always simplify the final answer (like reducing a fraction)

$\sqrt{2} \cdot \sqrt{6} = \sqrt{12}$

but $\sqrt{12} = \underbrace{\sqrt{4}}_{\text{square root of 4 is 2}} \cdot \sqrt{3} = 2\sqrt{3}$

So final answer: $2\sqrt{3}$

So, when mult. or diving radicals, IF they have the same index, you just go ahead and do it...

Ex: $\sqrt{6} \cdot \sqrt{5} = \sqrt{30}$ (there are no perfect square factors of 30 so you can't simplify)

$30 = 1 \cdot 3$
$2 \cdot 15$
$3 \cdot 10$
$5 \cdot 6$
} None with a perfect square root

Ex: $2\sqrt{5} \cdot 3\sqrt{15} = 6\sqrt{75}$

Both INSIDE the radical so $5 \cdot 15 = 75$

Both outside the radical so $2 \cdot 3 = 6$

Then simplify... think is there a factor of 75 that I can take the square root of?

$6\sqrt{75} = 6 \cdot \sqrt{25 \cdot 3}$
$= 6 \cdot 5\sqrt{3}$

since the $\sqrt{25} = 5$

FINAL ANSWER: $6 \cdot 5\sqrt{3} = 30\sqrt{3}$

Both outside so multiply

Ex: $3\sqrt[3]{15x^2} \cdot 2\sqrt[3]{3x^2y}$, $x \geq 0, y \geq 0$

these are restrictions on the variables so we don't accidentally try to $\sqrt{\ }$ a negative number

$= 6\sqrt[3]{45x^4y}$

⇒ Now simplify

① Does 45 have any perfect cube factors?

Perfect cubes are: 1, 8, 27, 64, ⋮

No.

② x^4 ...

$\sqrt[3]{x^4} = \sqrt[3]{x^3} \cdot \sqrt[3]{x}$

$x \cdot \sqrt[3]{x}$

$= 6 \cdot \sqrt[3]{45 \cdot x^3 \cdot x \cdot y}$

$= 6 \cdot x\sqrt[3]{45xy}$

FINAL ANSWER

③ The y can't be simplified

Ex: $\sqrt{10}(2 + 3\sqrt{7})$

· distribute the $\sqrt{10}$ into the bracket

$2\sqrt{10} + 3\sqrt{70}$

done

· inside the second radical, the $\sqrt{10} \cdot \sqrt{7} = \sqrt{70}$

· can we simplify $\sqrt{70}$? No

Ex: $(3\sqrt{7} + 2)(\sqrt{3} - 1)$

$3\sqrt{21} - 3\sqrt{7} + 2\sqrt{3} - 2$

Expand + simplify... FOIL

Check... any like terms to collect? Any radicals to simplify? No. then we are done!

FINAL ANSWER

$3\sqrt{21} - 3\sqrt{7} + 2\sqrt{3} - 2$

Dividing radicals:

① $\dfrac{\sqrt{80}}{\sqrt{10}} = \sqrt{8}$

$= \underline{\underline{2\sqrt{2}}}$

Final answer

· Same index so just divide

· Check... can I simplify $\sqrt{8}$?

$\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

② $\dfrac{10\sqrt[3]{30}}{20\sqrt[3]{10}} = \dfrac{1\sqrt[3]{3}}{2}$

$= \dfrac{\sqrt[3]{3}}{2}$

or $\frac{1}{2}\sqrt[3]{3}$

Final answer OR both OK

· divide the coefficients ($\frac{10}{20}$ simplifies to $\frac{1}{2}$)

· divide the radicands since they have the same index

$\sqrt[3]{\dfrac{30}{10}} = \sqrt[3]{3}$

③ $\dfrac{9\sqrt{432p^5} - 7\sqrt{27p^5}}{\sqrt{33p^4}}$, $p \geq 0$

the restriction is just there to restrict the variable p to POSITIVE values
– NOT part of th question

MEANS: $\dfrac{9\sqrt{432p^5}}{\sqrt{33p^4}} - \dfrac{7\sqrt{27p^5}}{\sqrt{33p^4}}$

Then ÷ $\dfrac{9 \cdot 12 \cdot \sqrt{p}}{\sqrt{11}} - \dfrac{7 \cdot 3 \cdot \sqrt{p}}{\sqrt{11}}$

(A) ① $\sqrt{\dfrac{432}{33}} = \sqrt{\dfrac{144}{11}} = \dfrac{12}{\sqrt{11}}$ reduce by ÷3

② and $\sqrt{\dfrac{p^5}{p^4}} = \sqrt{p}$

(B) ① $\sqrt{\dfrac{27}{33}} = \sqrt{\dfrac{9}{11}} = \dfrac{3}{\sqrt{11}}$

② $\sqrt{\dfrac{p^5}{p^4}} = \sqrt{p}$

$= \dfrac{108\sqrt{p}}{\sqrt{11}} - \dfrac{21\sqrt{p}}{\sqrt{11}}$

$= \dfrac{87\sqrt{p}}{\sqrt{11}}$

Final answer

These are LIKE terms so we can collect them

Conjugate pairs:

$x-5$ conjugate pair is $x+5$

$3+x \longrightarrow 3-x$

$\sqrt{x}-2 \longrightarrow \sqrt{x}+2$

Multiplying Conjugate pairs:

$(x-5)(x+5) = x^2 \underbrace{+5x-5x}_{\text{add up to } 0} - 25$

$= x^2 - 25$

$(3-x)(3+x) = 9+3x-3x-x^2$

$= 9-x^2$

There is a pattern!

$(a-b)(a+b) = a^2-b^2$

$(x-9)(x+9) = x^2-81$

The middle terms always add up to zero

Recall: $\sqrt{3}\cdot\sqrt{3} = \sqrt{9} = 3$

$\sqrt{5}\cdot\sqrt{5} = \sqrt{25} = 5$

$\sqrt{x}\cdot\sqrt{x} = \sqrt{x^2} = x$

so: $(\sqrt{x}+5)(\sqrt{x}-5) = \sqrt{x^2} - 25$

$= x - 25$ (the radical is gone!)

$(\sqrt{5}+\sqrt{x})(\sqrt{5}-\sqrt{x}) = \sqrt{25} - \sqrt{x^2}$

$= 5 - x$

$(\sqrt{x}-3)(\sqrt{x}+3) = x-9$ Follow the pattern.

Rationalizing the denominator:

→ Mathmaticians do NOT like radicals in the denominator. We like to move them to the numerator in a process called "rationalizing the denominator.

↑ making into a rational #

→ Remember we are allowed to multiply anything by 1 without changing it's value

But 1 can look funny...

$$1 = \frac{3}{3} = \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{x}}{\sqrt{x}} = \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} \ldots$$

Ex: ① $\frac{1}{\sqrt{3}}$

To turn the $\sqrt{3}$ into a rational #, we multiply it by itself → but we can't JUST change the bottom so we multiply by 1 in the form of: $\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{\sqrt{9}}$

$= \frac{\sqrt{3}}{3}$ No radical in the denominator... Tada!

Ex ② $\frac{20}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{20\sqrt{10}}{\sqrt{100}} = \frac{20\sqrt{10}}{10}$

↑ multiplying by 1 ... in the form of $\frac{\sqrt{10}}{\sqrt{10}}$

$= 2\sqrt{10}$

(since $\frac{20}{10} = 2$)

Ex ③ $\frac{-\sqrt{15}}{\sqrt{5x}}$, $m > 0$ (again, $m \geq 0$ is just a restriction on the variable)

Can we simplify inside? yes ... $\frac{15}{5} = 3$

so $\frac{-\sqrt{15}}{\sqrt{5x}} = \frac{-\sqrt{3}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{-\sqrt{3x}}{x}$ final answer

Ex ④ $\frac{\sqrt{7}}{\sqrt{3x}}$, $x > 0$

Nothing to simplify so multiply by $\frac{\sqrt{3x}}{\sqrt{3x}}$

$= \frac{\sqrt{7}}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}}$

$= \frac{\sqrt{21x}}{3x}$ Final answer

Ex: $\frac{5}{2+\sqrt{3}}$

→ Binomial in the denominator! → immediately put a bracket around it → it works as a team

$\frac{5}{(2+\sqrt{3})} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

Now multiply by a really funny 1 ...

the conjugate pair $\frac{(2-\sqrt{3})}{(2-\sqrt{3})}$

$= \frac{10-5\sqrt{3}}{4-3}$

Remember:

$(2+\sqrt{3})\cdot(2-\sqrt{3}) = 4-\sqrt{9}$
$= 4-3 = 1$

(the middle terms will cancel out)

$= \frac{10-5\sqrt{3}}{1} = \underline{\underline{10-5\sqrt{3}}}$

Ex: $\frac{\sqrt{3}+\sqrt{13}}{\sqrt{3}-\sqrt{13}}$ •

Binomial alert! ADD brackets!

$\frac{(\sqrt{3}+\sqrt{13})}{(\sqrt{3}-\sqrt{13})} \cdot \frac{(\sqrt{3}+\sqrt{13})}{(\sqrt{3}+\sqrt{13})}$

multiply by conjugate pair of the denominator

FOIL THE TOP

MULT THE BOTTOM (FOIL or use the conjugate pair pattern)

$= \frac{\sqrt{9}+\sqrt{39}+\sqrt{39}+\sqrt{169}}{3-13} = \frac{3+2\sqrt{39}+13}{-10}$

$= \frac{16+2\sqrt{39}}{-10}$

This answer is pretty good BUT I can factor the top...

→ take out a common factor of 2

$= \frac{2(8+\sqrt{39})}{-10}$

$= \frac{2(8+\sqrt{39})}{-10}$

$= \frac{(8+\sqrt{39})}{-5}$

$= -\frac{8+\sqrt{39}}{5}$

prettiest answer

BTW, it isn't WRONG to leave the − sign in the denom., but it is ugly so we usually move it out front or to the top.

Miss you guys!