

## Radicals AND RESTRICTIONS

- Restrictions state values that variables are NOT allowed to be. They are also referred to as
- You can NOT take the square root of a negative #, so, for example:

$\sqrt{x}$  ... the restriction is that  $x$  must be greater than or equal to zero. We write that as:  
 $x \geq 0$

$\sqrt{x+4}$  ... well, the radicand,  $x+4$ , must be greater than or equal to zero. So:

$$x+4 \geq 0 \quad \text{Now solve for } x$$

$$\quad \quad \quad -4 \quad -4$$

$x \geq -4$  this is the domain for  $x$  or the restriction on  $x$  so that the radicand is always positive

$$\sqrt{2x+7} = \sqrt{x-3}$$

restriction:                      restriction

$$2x+7 \geq 0$$

$$\quad -7 \quad -7$$

$$\frac{2x}{2} \geq \frac{-7}{2}$$

$$x \geq -\frac{7}{2}$$

$$x-3 \geq 0$$

$$\quad +3 \quad +3$$

$$x \geq 3$$

The OVERALL restriction for this question is  $x \geq 3$

If we used  $x \geq -\frac{7}{2}$ , then some of the #'s would cause problems in the other radical; so we use the most restrictive restriction

# SOLVING RADICAL EQUATIONS

## Type I : ONE RADICAL TERM

Restrictions

$$x \geq -\frac{1}{2}$$


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from:

$$2x+1 \geq 0$$

$$\begin{matrix} -1 & -1 \\ \hline 2x & \geq -1 \\ \hline x & \geq -\frac{1}{2} \end{matrix}$$

$$(\sqrt{2x+1})^2 = 3^2$$

$$2x+1 = 9 \quad \leftarrow$$

$$\begin{matrix} -1 & -1 \\ \hline 2x & = 8 \\ \hline x & = 4 \end{matrix}$$

\* State restrictions

STEPS

- ① Square both sides
- ② Then solve it
- ③ Check the answer

$$\sqrt{2(4)+1} \stackrel{?}{=} 3$$

$$\sqrt{8+1}$$

$$\sqrt{9} \checkmark = 3$$

Why we check:

$$(\sqrt{2x+1})^2 = \underline{\underline{(-3)^2}}$$

$$2x+1 = 9$$

$$\begin{matrix} -1 & -1 \\ \hline 2x & = 8 \\ \hline x & = 4 \end{matrix}$$

$$\sqrt{2x+1} = -3$$

$$\sqrt{2(4)+1} \stackrel{?}{=} -3$$

$$+\sqrt{9} \quad \cancel{=} -3$$

$$+\sqrt{9} = 3$$

"Extraneous Root"

↓  
Shows up in the math but leads to an impossible answer

## Type II Two Radicals, No other terms

$$\text{Ex 1 } (\sqrt{2-x})^2 = (\sqrt{x-2})^2$$

$$\begin{array}{r} 2-x \\ +2 \quad +x \end{array} = \begin{array}{r} x-2 \\ +x \quad -2 \end{array}$$

$$\begin{array}{l} 4 = 2x \\ x = 2 \end{array}$$

Check:

$$\sqrt{2-2} \stackrel{?}{=} \sqrt{2-2}$$

Square both sides!

then

Solve and check.

Restrictions:

$$\begin{array}{l} \sqrt{2-x} \\ 2-x \geq 0 \\ -2 \quad -2 \\ -x \geq -2 \\ -1 \quad -1 \\ \downarrow \\ x \leq 2 \end{array}$$

(When you divide both sides of an INEQUALITY by a negative #, the inequality changes direction)

$$\begin{array}{l} \sqrt{x-2} \\ x-2 \geq 0 \\ +2 \quad +2 \\ x \geq 2 \end{array}$$

So,  $x$  must be less than or equal to 2 AND greater than or equal to 2...

$$2 \leq x \leq 2 \quad \dots \text{ LOL, so I guess } x \text{ must be } \underline{\underline{2}}$$

Normally the restrictions would end up something like:

$$-3 \leq x \leq 2 \quad \text{or} \quad x \geq 7$$

... This question was unusual.