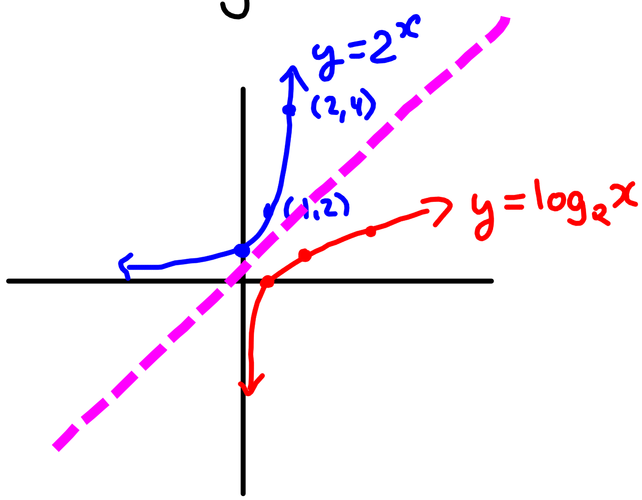
8.3 Log Functions



$\rightarrow y = 2^x$

INVERSE:

$x = 2^y$

$\rightarrow y = \log_2 x$ → $x > 0$

$$\lim_{x \to 0^+} \log_2 x = -\infty$$

(the y axis is an asymptote)

$$\lim_{x \to \infty} \log_2 x = \infty$$

So

$$\lim_{x \to 0^+} \log_b x = -\infty \quad \text{if } b > 1$$

$$\lim_{x \to \infty} \log_b x = \infty \quad \text{if } b > 1$$

8.4 Derivatives of Logs

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\ln(g(x)) = \frac{1}{g(x)}\cdot g'(x)$$

$$= \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}\log_b x = \frac{1}{x\ln b}$$

$$\frac{d}{dx}\log_b(g(x)) = \frac{1}{g(x)\ln b}\cdot g'(x)$$

$$\frac{d}{dx}b^x = b^x\ln b$$

$$\frac{d}{dx}b^{g(x)} = b^{g(x)}\cdot\ln b\cdot g'(x)$$

Ex 1 $y = x \ln x$

$f \cdot g' + f' \cdot g$

$y' = x \cdot \frac{1}{x} + 1 \cdot \ln x$

$y' = 1 + \ln x$

Ex2 $y = \ln(x^2 + 2x - 5)$

$y' = \frac{2x+2}{x^2+2x-5}$ $\qquad \frac{g'(x)}{g(x)}$

Ex3 $y = \ln(\cos x)$

$y' = \frac{1}{\cos x} \cdot -\sin x = -\frac{\sin x}{\cos x} = -\tan x$

Ex 4 $y = (\ln x)^4$

$y' = 4(\ln x)^3 \cdot \frac{1}{x} = \frac{4(\ln x)^3}{x}$

$$y = \ln\left[\frac{x}{\sqrt{x+1}}\right]$$

$$y' = \frac{1}{\frac{x}{\sqrt{x+1}}} \cdot \left[\frac{\sqrt{x+1}\cdot 1 - \frac{1}{2}(x+1)^{-1/2}\cdot x}{\left((x+1)^{1/2}\right)^2}\right]$$

$$= \frac{\frac{2\sqrt{x+1}\sqrt{x+1}}{2\sqrt{x+1}} - \frac{x}{2\sqrt{x+1}}}{\frac{(x+1)}{\frac{x}{\sqrt{x+1}}}}$$

$$y' = \frac{(2x+2-x)}{2\sqrt{x+1}(x+1)} \cdot \frac{\sqrt{x+1}}{x}$$

$$y' = \frac{x+2}{2x(x+1)}$$

$$y = \ln\left[\frac{x}{\sqrt{x+1}}\right] = \ln\frac{x}{(x+1)^{1/2}}$$

$$y = \ln x - \frac{1}{2}\ln(x+1)$$

$$y' = \frac{1}{x}\frac{2(x+1)}{(2(x+1))} - \frac{1}{2(x+1)}\frac{(x)}{(x)}$$

$$= \frac{2x+2-x}{2x(x+1)} = \frac{x+2}{2x(x+1)}$$