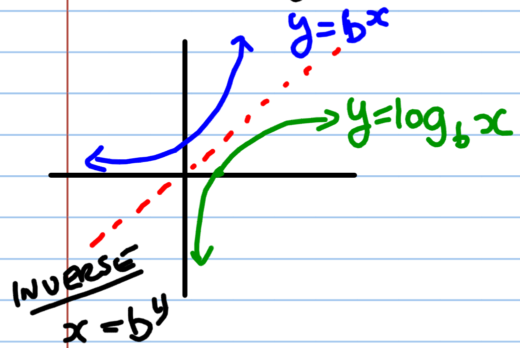
Limits of Log Fn's



$\log_b x = y$

$\star \lim_{x\to 0^+} \log_b x = -\infty$

$\lim_{x\to 0^+} \ln x = -\infty$ ($\log_e x$)

$\star \lim_{x\to \infty} \log_b x = +\infty$

DERIVATIVES OF LOG Fn's

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \ln(f(x)) = \frac{1}{f(x)} \cdot f'(x)$

$= \frac{f'}{f}$

$\frac{d}{dx} \log_b x = \frac{1}{x \ln b}$

$\frac{d}{dx} b^x = b^x \ln b$

Ex

$y = \ln(2x^3) \qquad y' = \frac{6x^2}{2x^3} = \frac{3}{x}$

$y = \ln(\cos x) \qquad y' = \frac{-\sin x}{\cos x} = -\tan x$

Ex $y = \ln\left(\frac{x}{\sqrt{x+1}}\right)$

$$y' = \frac{\dfrac{(\sqrt{x+1})\cdot 1 - \left(\frac{1}{2}(x+1)^{-1/2}\cdot x\right)}{(\sqrt{x+1})^2}}{\dfrac{x}{\sqrt{x+1}}}$$

$$= \frac{\dfrac{\frac{2\sqrt{x+1}}{2\sqrt{x+1}}(\sqrt{x+1}) - \frac{x}{2\sqrt{x+1}}}{(x+1)}}{\dfrac{x}{\sqrt{x+1}}} = \frac{\dfrac{\dfrac{2(x+1)-x}{2\sqrt{x+1}}}{(x+1)}}{\dfrac{x}{\sqrt{x+1}}}$$

$$\frac{x+2}{2\cancel{\sqrt{x+1}}(x+1)} \cdot \frac{\cancel{\sqrt{x+1}}}{x}$$

$$\boxed{\frac{x+2}{2x(x+1)}}$$

$$y = \ln\left(\frac{x}{\sqrt{x+1}}\right) = \ln x - \underline{\underline{\frac{1}{2}}} \ln(x+1)$$

$$\downarrow$$

$$\frac{1}{x} - \frac{1}{2} \cdot \frac{1}{x+1}$$

$$\frac{2(x+1)}{2(x+1)} \frac{1}{x} - \frac{1}{2(x+1)} \frac{(x)}{(x)}$$

$$\frac{2x+2-\cancel{x}}{2x(x+1)} = \frac{x+2}{2x(x+1)}$$

$$f(x) = \log(3x+1)^4 = 4\log(3x+1)$$

$$f' = 4 \cdot \frac{3}{(3x+1)\ln 10} = \frac{12}{(3x+1)\ln 10}$$

$$f' = \frac{4\cancel{(3x+1)}^{3} \cdot 3}{(3x+1)^{\cancel{4}}\ln 10}$$

$$\ln y = \ln\left[\frac{e^x\sqrt{x^2+1}}{(x^2+2)^3}\right] \longleftarrow$$

Logarithmic Differentiation

8.6

$$\frac{d}{dx}\ln y = \frac{d}{dx}\left(\cancel{\ln e^x}\,x + \frac{1}{2}\ln(x^2+1) - 3\ln(x^2+2)\right)$$

$$\frac{1}{y}\cdot y' = 1 + \frac{1\cdot\cancel{2}x}{\cancel{2}(x^2+1)} - \frac{3(2x)}{x^2+2}$$

y

$$\boxed{y' = \left[1 + \frac{x}{x^2+1} - \frac{6x}{x^2+2}\right]\frac{e^x\left(\sqrt{x^2+1}\right)}{(x^2+2)^3}}$$