INTERNATIONAL ECONOMIC REVIEW
Vol. 30, No. 4, November 1989

MORAL HAZARD AND LIMITED LIABILITY: IMPLICATIONS FOR THE THEORY OF THE FIRM

BY JAMES A. BRANDER AND BARBARA J. SPENCER[1]

This paper presents a theory of the firm based on moral hazard in the provision of effort by an owner-manager who borrows money in financial markets under conditions of limited liability. We examine the relationship between the financial structure of the firm and the effort and output decisions of the owner-manager. Results are reported concerning the determination of the firm's optimal financial structure, and concerning the positive and normative implications of financial structure for pure competition and monopoly. We also identify a strategic advantage from equity finance under Cournot oligopoly.

1. INTRODUCTION

The "agency" view of the firm, based on the idea that delegation of responsibility under asymmetric information is fundamental to the theory of the firm, has become very influential in industrial organization and in the business disciplines. In this paper we present a simple yet rigorous model structure capturing the agency relationship between the owner-manager (or inside equity holders) of a firm and the bondholders under conditions of limited liability. One objective is to incorporate a meaningful financial decision into the traditional theory of (output) market structure, particularly the theory of pure competition and monopoly. We also wish to contribute to the growing literature showing the relevance of market structure for financial decisions, and to explore generally the linkages between financial structure, managerial effort, bankruptcy, and the product market. We would not describe our model as a "canonical" version of the agency approach to the integration of financial and real decisions, but we hope it is a step in that direction.

We assume that the firm is controlled by an owner-manager whose effort is private information and therefore cannot be observed by creditors. In the presence of limited liability this creates an agency problem of the moral hazard type. An important aspect of the model is that the owner-manager must finance the production process (through commitment of his own equity and through borrowing) before production is actually carried out and the effort decision is made. As a result, output levels will depend on the financial structure of the firm, as will the risk of bankruptcy. We solve for the owner-manager's optimal selection of borrowing and equity, and we report the conditions under which the firm will be completely equity financed, completely debt financed, or partially financed by both debt and equity.

[1] This paper has been presented to very helpful audiences at the Japan-U.S. Conference on Game Theory: Applications to Economics and Finance, New York University, April, 1988, and in the UBC Finance Workshop. We thank Kose John, Julianne Nelson, Avri Ravid, Venk Sadanand, and two anonymous referees for particularly helpful comments.

We are able to make some interesting comparisons between monopoly and competition, and we identify a strategic advantage of equity finance in oligopoly. We also examine the welfare implications of the introduction of borrowing possibilities for the standard "all-equity" firm normally assumed in the theory of industrial organization.

Basic references on moral hazard include Pauly (1974), Holmstrom (1979), and Shavell (1979), and a widely cited application of moral hazard to sharecropping is contained in Stiglitz (1974). Our paper differs from most treatments of moral hazard because of the introduction of limited liability. The one general treatment of moral hazard that does deal with limited liability is Sappington (1983). An important difference between that paper and ours is that Sappington assumes that uncertainty is resolved before the agent (owner-manager) undertakes his action, whereas in our model, uncertainty is resolved after the agent has acted. This difference has significant consequences. For example, in Sappington (1983) agents will offer no effort in bad states of the world, because they know that their return will be zero in any case. In our model, the agent always offers some effort.

The importance of agency considerations for the theory of the firm was persuasively argued by Williamson (1975) and Jensen and Meckling (1976), and the agency cost approach to financial structure figures prominently in a number of recent papers, including Fama (1980), John and Nachman (1985), Darrough and Stoughton (1986), and Williams (1987). Recent work focusing on the linkage between financial decisions and product markets includes Titman (1984), Brander and Lewis (1986, 1988), Maksimovic (1986), Allen (1986), and Poitevin (1988). A very useful survey of the finance/output linkage in Ravid (1988).

An outline of the paper is as follows. Section 2 sets out the basic model. Sections 3 and 4 analyze the effort and output decisions of the owners of the firm, given the firm's financial structure. Section 5 considers the determination of financial structure, and Section 6 compares the output market behavior of leveraged firms with corresponding all equity firms. Sections 2 through 6 restrict attention to the cases of competition and monopoly. Section 7 discusses oligopoly. Section 8 contains concluding remarks. An Appendix contains proofs of propositions.

2. MODEL SPECIFICATION

The central decision-maker in our model is the owner-manager of the firm, who is taken to represent inside equity holders.[2] This owner contributes some of his own financial capital, E, referred to as equity capital, and borrows an amount B from the bond market to finance the firm's production. Output y is produced using the effort of the owner-manager, denoted e, and other factors, represented by a single aggregate, denoted I. The market price of this aggregate factor is normalized to equal one, so I represents both the physical quantity and the cost of the factor. It is these costs I, referred to as "investment," that must be financed by debt and equity:

[2] We abstract from the existence of "outside" equity holders: outside interests who hold equity but do not observe the owner's effort or contribute their own.

$$B + E = I. \tag{2.1}$$

In addition, output depends on a random variable z, assumed to be distributed according to density function $f(z)$ over the interval [0, 1], with cumulative distribution function $F(z)$. Using p to denote price, the revenue of the firm, R can be written as follows:

$$R(e, I, z) = p(Y)y(e, I, z), \tag{2.2}$$

where Y represents industry output, which is equal to firm output in the case of monopoly. The production function $y(e, I, z)$ is assumed to be increasing in z, indicating that higher values of z represent better states of the world, and is also increasing in e and I. We impose the following conditions on $R(e, I, z)$. (Subscripts denote partial derivatives.)

$$R_e \equiv (p + yp')y_e > 0 \quad R_I \equiv (p + yp')y_I > 0 \quad R_z \equiv (p + yp')y_z > 0. \tag{2.3}$$

For pure competition, $p' = 0$ as perceived by the individual firm, and the conditions in (2.3) follow directly from the assumption that output is increasing in its arguments. For other market structures, conditions (2.3) require that the firm is always in a region where marginal revenue is strictly positive. We also assume diminishing marginal productivity in e and I and that cross effects between e, I, and z are (weakly) positive.

$$y_{ee} < 0 \quad y_{II} < 0 \quad y_{eI} \geq 0 \quad y_{ez} \geq 0 \quad y_{Iz} \geq 0. \tag{2.4}$$

To borrow B, the firm issues debt with face value D. For a given debt obligation, D, investment, I, and effort level, e, there is a critical state of the world, z^*, at which the firm is just able to pay its debts. Thus, for positive levels of debt,

$$R(e, I, z^*) - D = 0. \tag{2.5}$$

Equation (2.5) defines z^* as a function of e, I, and D:

$$z^* = z^*(e, I, D). \tag{2.6}$$

For states of the world below z^*, the firm is bankrupt and the creditors of the firm become the residual claimants. For states of the world above z^*, the firm pays D to the creditors, and the owner-manager is the residual claimant. For any strictly positive value of D, the critical state, z^*, is assumed to be strictly between 0 and 1, ensuring that there is some positive probability of bankruptcy. If D is equal to zero then there is no possibility of bankruptcy and $z^* = 0$.

The effects of changes in e, I, and D on z^* are obtained from total differentiation of (2.5), and signed using conditions (2.3), yielding the results that, other things equal, the critical state, z^*, at which the firm is just able to pay its debts, is decreasing in effort, decreasing in investment, and increasing in debt:

$$z_e^* = -R_e/R_z < 0 \quad z_I^* = -R_I/R_z < 0 \quad z_D^* = 1/R_z > 0. \tag{2.7}$$

Holding debt obligations constant, increasing either e or I increases expected revenue and means that the firm will be solvent in a wider range of states of the world. Increasing D, holding e and I constant, increases the probability of bankruptcy.

There are two decision stages. In stage 1, the owner-manager chooses the levels of equity E and borrowing B that will be used to finance investment I. The face value of debt, D, is simultaneously determined in the bond market. In stage 2, taking the stage 1 borrowing and equity decisions as given, the owner decides on the level of (unobservable) effort, after which random variable z is realized, output is produced, and payoffs are made. This sequential structure reflects what we take to be important aspects of most production processes: the firm requires "working capital" to finance production, and the ultimate revenues from production are uncertain at the time managerial effort is committed to the production process.

A sequential rationality constraint is imposed on the equilibrium. In particular, in stage 1 bondholders correctly anticipate the effect of the firm's financial structure on the level of managerial effort. Creditors cannot write financial contracts that are contingent on effort or on the state of the world, as both are private information to the owner-manager, who cannot credibly promise to do anything other than choose his effort to maximize his own utility in stage 2.

The expected utility of the owner-manager in stage 2 is

$$U(e; I, D, E) = \int_{z^*}^{1} (R(e, I, z) - D)f(z)\, dz - G(e) - v(E), \tag{2.8}$$

where z^* is given by (2.6). The function $G(e)$ represents the owner's disutility of effort and $v(E)$ is the owner's opportunity cost of equity. Marginal opportunity cost $v'(E)$ is taken to be increasing in E, reflecting the idea that the owner's personal wealth is limited and that an increase in equity capital is achieved by transferring wealth from progressively higher valued consumption or investment alternatives. Expression (2.8) incorporates the idea that the owner focuses only on expected revenue over non-bankrupt states of the world. Note also that the cost of production, I, does not enter expression (2.8) directly, reflecting the point that it is not the direct cost of buying inputs that is relevant, but the cost to the owner of raising amount I.

Debt, D, is determined in stage 1 in a competitive bond market, in which participants are taken to be risk neutral. Bondholders are assumed to have a completely safe asset available on which they can earn interest at rate i. Because the debt market is competitive, the risky asset B must offer the same expected value as investing B at interest rate i in the safe asset.

$$B(1 + i) = D(1 - F(z^*)) + \int_{0}^{z^*} R(e, I, z)f(z)\, dz. \tag{2.9}$$

The first term on the right hand side of equation (2.9) represents the value associated with the prospect of being paid off in full, which occurs when the firm is solvent ($z \geq z^*$). ($F(z^*)$ is the probability of bankruptcy.) The second term represents the revenues of the firm in bankrupt states, when creditors become the residual claimants. The face value, D, exceeds $B(1 + i)$, while the return in a bankrupt state is something less than $B(1 + i)$. Sequential rationality implies that the values of e and $z^*(e, I, D)$ that appear in expression (2.9) are the actual values that will be determined in stage 2. Equation (2.9) implicitly defines[3] D as a function of B and E.

$$D = D(B, E). \tag{2.10}$$

One might ask why potential creditors do not directly exploit the profitable opportunities identified by the firm, avoiding the costs of moral hazard. One possible reason is that the market opportunities may reflect information or opportunities that are not available directly to creditors. Alternatively, it is possible that each creditor has financial resources that are small compared to the investment required to achieve efficient levels of production. If one creditor tried to raise money by borrowing from other creditors, he would face the same problem faced by the owner in our model. Because of this unavoidable market failure, our model represents a departure from the Arrow-Debreu world of perfect markets, even when firms are price takers in the output market.

3. EFFORT AND OUTPUT CHOICES

In stage 2, the owner-manager chooses e to maximize his expected utility given by (2.8) taking I, D, and E as given from stage 1. Using (2.5) the first order condition reduces to

$$U_e(e; I, D, E) = \int_{z^*}^{1} R_e(e, I, z)f(z)\,dz - G'(e) = 0. \tag{3.1}$$

Equation (3.1) indicates that, at the owner-manager's optimum, the marginal return from effort over non-bankrupt states must be just equal to the marginal disutility of effort. Letting R_e^* denote $R_e(e, I, z^*)$, the second order condition associated with the choice of effort is

$$U_{ee} = \int_{z^*}^{1} R_{ee}(\cdot)f(z)\,dz - R_e^* f(z^*)z_e^* - G''(e) < 0, \tag{3.2}$$

which we assume holds globally over the region of interest.

Adam Smith (1976, Vol. 2, 264–265) made an early observation about moral hazard: "the managers of other people's money . . . cannot be well expected . . . [to] watch over it with the same anxious vigilance . . . [they] watch over their own. Like the stewards of a rich man, they are apt to consider attention to small matters as not for their master's honour, and very easily give themselves a dispensation from having it."

To show this result formally in our model, assume that consumer utility, denoted

[3] In order for the conditions of the implicit function theorem to be met in this case, we must assume that $1 - F(z^*) + e_D \int_0^{z^*} R_e f(z)\,dz \neq 0$. This holds for all cases of interest.

ϕ, is quasilinear in Y and expenditure on other goods, m:[4] $\phi = u(Y) + m$. Assuming that there are n identical firms so that $Y = ny$, expected aggregate welfare, W, is the expected utility from good Y less production and net financing costs (including the forgone interest iB on borrowed funds):

$$W(e, B, E, n) = \int_0^1 u(Y)f(z)\,dz - n[G(e) + v(E) + B(1+i)]. \tag{3.3}$$

At the welfare optimum, e satisfies the first order condition

$$W_e(\cdot) = n\left[\int_0^1 py_e\, f(z)\,dz - G'(e)\right] = 0. \tag{3.4}$$

Since $R_e \leq py_e$ (from (2.3)), and z^* is positive (given positive debt), a comparison of (3.4) with (3.1) indicates that W_e is strictly positive at the effort level chosen by owners. Assuming that second order condition $W_{ee} < 0$ holds over the relevant region, Proposition 1 follows.

PROPOSITION 1. *For any given level of I and any positive level of D, owners will use less than the socially efficient level of effort.* □

The next issue to be addressed is the effects of changes in investment I and debt D on the level of managerial effort chosen in stage 2. First order condition (3.1) implicitly defines

$$e = \psi(I, D). \tag{3.5}$$

The effects of I and D on effort are given by the partial derivatives of ψ, and denoted e_I and e_D respectively. From total differentiation of (3.1) with respect to e and D,

$$e_D = -U_{eD}/U_{ee} = f(z^*)R_e(e, I, z^*)z^*_D/U_{ee} < 0. \tag{3.6}$$

From (2.3), (2.7), and (3.2), an increase in debt, D, holding investment constant, reduces the owner's effort. This is because an increase in D increases the chance of bankruptcy, other things equal, reducing the range of states of nature in which the owner receives a return from his effort.

Similarly, totally differentiating (3.1) with respect to e and I yields:

$$e_I = -U_{eI}/U_{ee} = -\left[\int_{z^*}^1 R_{eI}\, f(z)\,dz - f(z^*)R_e(e, I, z^*)z^*_I\right]\Big/ U_{ee}. \tag{3.7}$$

Using (2.3), (2.7), and (3.2), it follows that $e_I > 0$ if $R_{eI} > 0$. Under pure competition, we have $R_{eI} > 0$ (from (2.4)), implying that investment tends to

[4] This commonly used specification assumes away various theoretical issues, including income effects, aggregation problems, and second-best problems associated with distortions elsewhere in the economy. Use of quasi-linear utility reduces welfare measurement to partial equilibrium surplus measures.

increase effort because more investment raises the marginal product of effort. In addition, the range of states that are relevant to the owner expands, reinforcing his incentive to increase effort. Under monopoly, from (2.3)

$$R_{eI} = (p + yp')y_{eI} + y_e(2p' + yp'')y_I. \tag{3.8}$$

Since the second term of (3.8) is negative, it is possible that $R_{eI} < 0$. It is therefore possible that under monopoly (or under imperfect competition generally) effort may fall as investment rises.

Output also depends directly on I and D. From (2.2) and (3.5), $y = y(e, I, z)$ where $e = \psi(I, D)$. The effects of I and D on output, taking into account the induced changes in e, are:

$$dy/dI = y_I + y_e e_I \qquad dy/dD = y_e e_D < 0. \tag{3.9}$$

Expression (3.9) implies that an increase in debt always reduces output. It also implies that an increase in investment increases output if e_I is positive, as is the case with perfect competition. Even if e_I is negative, as it may be under monopoly, one would normally expect the direct effect of investment on output to dominate, leading to an overall increase in output. Proposition 2 shows that this is the case if investment and effort levels are independent of z ($y_{Iz} = y_{ez} = 0$). (See the Appendix for proofs of propositions.)

PROPOSITION 2. *i) An increase in debt, holding investment constant, causes effort and output to fall:* $e_D(I, D) < 0$ *and* $dy/dD < 0$. *ii) Under pure competition, an increase in investment, holding debt constant, causes effort and output to rise.* ($e_I(I, D) > 0$ *and* $dy/dI > 0$). *This also holds for monopoly if* $R_{Ie} \geq 0$. *iii) Under monopoly, an increase in investment causes output to rise if* $y_{Iz} = y_{ez} = 0$: $dy/dI > 0$. (*This holds even if* $R_{Ie} < 0$ *and* $e_I < 0$.) □

Hicks (1935) wrote that "the best of all monopoly profits is a quiet life." Proposition 3 reports a similar idea: a monopoly owner will put in less effort than a comparable competitive owner. There are at least two reasonable ways to make this comparison: by comparing a monopoly firm and a corresponding single competitive firm, or by comparing a monopoly firm with a corresponding competitive industry. Both comparisons are reported in Proposition 3.

PROPOSITION 3. *i) Holding borrowing and equity (and therefore investment) the same for both firms, the owner of a monopoly firm uses less effort than the owner of a corresponding competitive firm. ii) Assume that a monopoly firm uses the same level of borrowing and equity as a corresponding competitive industry. Assume further that output is homogeneous of degree* 1 *in* e *and* I. *The monopolist then uses less effort than does the competitive industry.* □

4. EFFECTS OF BORROWING AND EQUITY ON OUTPUT

Investment and debt are themselves endogenous variables determined by the underlying choices of borrowing B and equity investment E. Substituting (2.1) and (2.10) into (3.5) we obtain effort as a function of the financial variables B and E.

$$e = \psi(B + E, D(B, E)) \equiv e(B, E). \tag{4.1}$$

The partial derivatives of the function $e(B, E)$ are

$$e_B = e_I + e_D D_B \qquad e_E = e_I + e_D D_E. \tag{4.2}$$

We first examine the effect of increased borrowing on the debt owed by the firm. Total differentiation of (2.9) (using (2.5)) yields

$$D_B(B, E) = \left(1 + i - \int_0^{z^*} (dR/dB)f(z)\,dz\right) \Big/ (1 - F(z^*)) \tag{4.3}$$

where $dR/dB \equiv R_I + R_e e_B$.

If debt were completely risk free, then a one dollar increase in borrowing would simply cause debt to increase by $(1 + i)$. In the case of risky debt, however, we would expect the marginal cost of borrowed capital to exceed the risk free rate (i.e. $D_B > 1 + i$). It can be shown that this is the case for all B provided that the marginal revenue associated with extra borrowing is decreasing in B (i.e. that $d^2R/(dB)^2 < 0$ for all $B > 0$). A useful explicit form for D_B is obtained by substituting (4.2) into (4.3), yielding

$$D_B = \left[1 + i - \int_0^{z^*} (R_I + R_e e_I)f(z)\,dz\right] \Big/ a \tag{4.4}$$

where $a = 1 - F(z^*) + e_D \int_0^{z^*} R_e f(z)\,dz$. We assume that $a > 0$. (In Section 5 it is demonstrated that a must be positive if E and B are chosen optimally by the owner-manager.)

Similarly, from (2.9), the effect of a change in equity on the cost of debt is

$$D_E(B, E) = -\int_0^{z^*} (dR/dE)f(z)\,dz/(1 - F(z^*)) \tag{4.5}$$

where $dR/dE = R_I + R_e e_E$. Then, substituting (4.2) into (4.5) we obtain

$$D_E(B, E) = -\int_0^{z^*} (dR/dI)f(z)\,dz/a \tag{4.6}$$

where $dR/dI = R_I + R_e e_I = (p + yp')\,dy/dI$ from (2.3) and (3.9). Since $a > 0$, it follows that D_E is negative provided dy/dI is positive. Thus an increase in E will reduce the cost of debt (for a given level of B), if an increase in investment leads to an increase in output. From Proposition 2, this holds under pure competition, and is usually the case for monopoly as well. Higher levels of equity increase the security of debt and therefore lower the required risk premium for a given level of borrowing.

We are now able to discuss the effects of changes in B and E on the owner's effort and on output. From (2.2) and (4.1), $y = y(e, I, z)$ where $e = e(B, E)$, so that

$$dy/dB = y_I + y_e e_B \qquad dy/dE = y_I + y_e e_E \tag{4.7}$$

where $e_B = e_I + e_D D_B$ and $e_E = e_I + e_D E_E$.

The effect of an increase in borrowing on effort and output is ambiguous under pure competition. Borrowing more raises investment directly, and this tends to increase effort and output. However, borrowing more also raises the firm's debt and raises the risk premium required by debtholders. This increase in debt tends to lower effort and output because the owner becomes a residual claimant over a smaller range of states of nature. The same issues arise under monopoly, with the additional ambiguity arising from the possibility that e_I might be negative.

An increase in equity capital increases both effort and output if markets are competitive. This follows since $e_D < 0$ and $e_I > 0$ (from Proposition 2), and $D_E < 0$. The monopoly case is again ambiguous because e_I and e_E may be negative, but output must rise in the monopoly case whenever $dy/dI > 0$. Despite the ambiguities in the effects of E and B in isolation, more definitive (and more interesting) results can be obtained for "balanced budget" changes in equity and borrowing.

PROPOSITION 4. *Under pure competition or monopoly, if an owner-manager substitutes borrowed funds for equity, maintaining investment fixed, then: i) the effort of the owner declines, ii) the firm's output falls, and iii) the probability of equity at bankruptcy increases.* □

5. THE CHOICE OF FINANCIAL STRUCTURE

In response to the Modigliani and Miller (1958) assertion that, under certain conditions (corresponding roughly to "perfect markets"), financial structure is irrelevant to the value of the firm, a large literature has developed, focusing on departures from perfect markets that are capable of providing a determinate solution for financial structure. Four distinct approaches can be distinguished. One approach, illustrated by Kraus and Litzenberger (1973), examines the tradeoff between the tax advantages of debt and the greater likelihood of suffering bankruptcy costs as debt rises. A second approach, associated with Ross (1977) and Leland and Pyle (1977), argues that financial structure acts a signal of the firm's private information. The third well-known approach, associated with Jensen and Meckling (1976), stresses the "agency cost" effects of debt and equity, and the fourth approach introduces the role of financial structure in creating strategic commitments in output markets. This paper is in the spirit of the agency cost approach, although strategic output market considerations may also enter, if the output market is oligopolistic (see Section 7).

Substituting expressions (2.1), (2.10), and (4.1) into (2.8), we can define the expected utility of the owner-manager as a function of the levels of B and E set in stage 1.

$$\hat{U}(B, E) = U(e(B, E), B + E, D(B, E), E). \tag{5.1}$$

Borrowing and equity are assumed to be chosen simultaneously in stage 1, although there is no significant difference if equity is chosen prior to the borrowing decision. First order conditions associated with stage 1 maximization are as follows.

(5.2) $$\hat{U}_B = U_e e_B + U_I + U_D D_B = 0,$$

(5.3) $$\hat{U}_E = U_e e_E + U_I + U_D D_E + U_E = 0.$$

Recalling that $U_e = 0$ from (3.1), and using (2.5) and (2.8) we obtain

(5.4) $$\hat{U}_B = \int_{z^*}^{1} (R_I - D_B) f(z)\, dz \leq 0 \quad (=0 \text{ if } B > 0),$$

(5.5) $$\hat{U}_E = \int_{z^*}^{1} (R_I - D_E) f(z)\, dz - v'(E) \leq 0 \quad (=0 \text{ if } E > 0).$$

We assume that second order conditions are also satisfied:

(5.6) $$\hat{U}_{BB} < 0 \qquad \hat{U}_{EE} < 0 \qquad \hat{U}_{BB}\hat{U}_{EE} - (\hat{U}_{EB})^2 > 0.$$

First order conditions (5.4) and (5.5) can be used to derive:

PROPOSITION 5. (*i*) *If the owner has access to equity capital at an opportunity cost equal to the competitive market riskless rate of interest* ($v(E) = (1 + i)E$), *then* $B = 0$ *and the firm will be entirely equity financed.* (*ii*) *Let the utility maximizing level of equity at* $B = 0$ *be denoted* E^0. *If* $v'(E^0)$, *the marginal opportunity cost of* E^0, *exceeds* $1 + i$, *then the owner will take on positive amounts of debt.* □

Assuming private information, Proposition 5 makes the basic point that debt financing can only be expected if the owner is wealth constrained. If the scale of the investment opportunities is small compared to the personal wealth of the owner-manager, then we should expect the firm to be entirely equity financed. More concretely, if the owner manager has assets earning only the safe market rate of return, then he should use those assets in the firm before trying to raise debt. Once the owner has exhausted his low opportunity cost sources of equity, and must consider selling the family house, or liquidating a holding in some other very promising investment, then he will pay the premium necessary to borrow financial capital.

The formal condition for borrowing to take place is expressed in Proposition 5(ii). If the marginal opportunity cost of equity finance, $v'(E)$ rises above $1 + i$ before all investment opportunities with rates of return above $1 + i$ are exhausted, then the owner will use some debt finance, because the marginal cost of the first dollar of debt finance is itself only $1 + i$. Considering the other corner solution, it is possible that the firm could be entirely debt financed. This would occur if $v'(E)$ were very high even at $E = 0$. An extreme example is the case of an owner who has essentially no discretionary personal wealth except for his access to profitable production opportunities which, by assumption, are private knowledge and cannot profitably be sold directly. In general, however, we expect that owners will have some discretionary wealth with an opportunity cost equal to the competitive market rate. The normal solution, therefore, would involve an interior solution for both debt and equity. The exact composition of debt and equity is determined by the tradeoff between the agency costs of debt finance and the rising marginal opportunity cost of equity finance.

Using expressions (5.4) and (5.5), an internal solution for debt and equity satisfies (5.7).

$$\int_{z^*}^{1} R_I\, f(z)\, dz = D_B(1 - F(z^*)) = D_E(1 - F(z^*)) + v'(E). \tag{5.7}$$

The second equality in (5.7) indicates that in equilibrium the owner chooses the debt/equity ratio to equalize the marginal cost to the firm of debt and equity finance. The effects of borrowing and equity on the cost of borrowing, D, is only relevant in nonbankrupt states, whereas the owner incurs the private opportunity cost of his equity in all states of the world. The first term of (5.7) indicates that this common marginal cost is set equal to the marginal value of investment over non-bankrupt states. From (5.7), (4.4), and (4.6), it follows that

$$v'(E) = (1 + i)(1 - F(z^*))/a, \tag{5.8}$$

where $a = 1 - F(z^*) + e_D \int_0^{z^*} R_e f(z)\, dz$, which implies that a is strictly positive as claimed in Section 4 of the paper.

Another useful way of viewing first order conditions (5.4) and (5.5) is to use (4.3) and (4.5) to obtain

$$\hat{U}_B = \int_0^1 R_I\, f(z)\, dz - (1 + i) + e_B \int_0^{z^*} R_e f(z)\, dz = 0, \tag{5.9}$$

$$\hat{U}_E = \int_0^1 R_I\, f(z)\, dz - v'(E) + e_E \int_0^{z^*} R_e f(z)\, dz = 0. \tag{5.10}$$

The first term of (5.9) shows the value of the increased investment associated with an increase in borrowing. The opportunity cost of borrowed funds is subtracted in the second term. These two terms together would constitute the entire first order condition in the absence of agency costs. In addition, however, there is a third term, which reflects the additional agency cost associated with the idea that increases in borrowing tend, in themselves, to induce the owner-manager to adjust his or her effort level. Expression (5.10) has a similar interpretation.

6. FINANCIAL STRUCTURE, OUTPUT MARKETS AND WELFARE

One issue of fundamental importance to our analysis is the question of how the properties of our model compare with models that surpress or eliminate issues of financial structure. Rather than make the trivial comparison with the perfect markets world of Modigliani and Miller, a more interesting basis for comparison is a world in which the moral hazard problem created by unobservable effort is present, but in which the debt market is absent. One can then ask what effect the introduction of debt markets and the introduction, therefore, of a meaningful financial structure decision, has on effort, output, and welfare. The net effect of the introduction of a debt market is ambiguous in general. The firm will substitute debt for equity, which tends to reduce effort and output, but the firm will also increase investment, which tends to raise effort and output. We show, however, that welfare increases in the competitive case. Propositions 6 and 7 summarize the main results. In Proposition 6(ii) we use the following condition:

$$\hat{U}_{EE} - \hat{U}_{EB} < 0. \tag{6.1}$$

This condition is closely related to (but slightly stronger than) second order condition (5.6).

PROPOSITION 6. *Assume that $v'(E^0) > 1 + i$. Then, the introduction of the possibility of borrowing by an all equity firm, i) decreases effort and output for any given investment level, and ii) results in a net increase in investment if (6.1) holds.* □

PROPOSITION 7. *(i) A purely competitive industry chooses the second best[5] efficient level of debt and equity, given moral hazard in the choice of effort. (ii) Provided $v'(E^0) > (1 + i)$, the introduction of debt into a purely competitive all equity industry increases welfare.* □

In the case of monopoly, the welfare effects of introducing credit markets are ambiguous. Under monopoly, the owner chooses too little investment and produces too little output compared to the second best solution. Although the introduction of credit markets ensures that a monopoly owner will increase investment, which is a move in the right direction, it is possible that the owner's effort will fall sufficiently that overall welfare is reduced. In effect, the presence of credit markets allows a monopolist to further exploit monopoly power with respect to his effort choice.

7. OLIGOPOLY: THE COMMITMENT EFFECT OF FINANCIAL STRUCTURE

So far we have restricted attention to the output market structures of perfect competition and monopoly. The implications of moral hazard for the relation between financial structures and the output market are, however, particularly interesting in the case of Cournot oligopoly. We do not set out the algebra for the oligopoly case here, but the basic intuition is as follows. Proposition 4 implies that a substitution of equity for debt causes a monopoly to increase output. It follows that a corresponding Cournot oligopolist would experience an outward shift in its

[5] The "first-best" solution is the solution that would arise if social welfare were maximized with respect to all choice variables: borrowing, equity, effort, and the number of firms, as if there were an altruistic regulator with full information and full control. The "second-best" solution arises if the social welfare maximizing levels of debt, equity, and number of firms are chosen, given the constraint that the owner-manager will choose his effort level to maximize his own welfare, as if an altruistic regulator could observe and control financial decisions and entry decisions, but not effort.

reaction function as a result of substituting equity for debt. This leads, under normal regularity conditions, to an equilibrium increase in own output, a decline in the output of rivals, and an increase in industry output as a whole. In effect, financial structure can be used to create a commitment to an aggressive output market strategy.[6] Specifically, substituting equity for debt has the effect of commiting the owner manager of a firm to providing more effort. This is a credible threat that will act to deter rivals, providing a strategic motive for using more equity than would otherwise be the case. This might be referred to as the effort effect of equity.[7]

8. CONCLUDING REMARKS

This paper offers an "agency" theory of the firm in which an owner-manager seeks to borrow money from creditors who cannot observe the effort provided by the owner-manager. Under limited liability, a moral hazard problem arises, creating a linkage between financial structure and the output market. As indicated by Proposition 4, the substitution of borrowed funds for equity investment induces less effort and output from the firm, and the probability of bankruptcy rises. We also show that, for a given financial structure, a monopolist will use less effort than a competitive owner. Financial structure takes on a strategic role in oligopoly markets precisely because it affects the firm's output stance.

The main welfare results are reported in Propositions 1 and 7. Proposition 1 shows that the very presence of debt will induce owner-managers to use less than the socially efficient level of effort and produce less than the "first best" level of output for any given level of investment in the firm. However, Proposition 7 shows that pure competition does achieve second best efficiency in the sense that welfare is maximized given the moral hazard constraint. Also, although the introduction of credit markets into the 100% equity would normally assumed in industrial organization may reduce output, it is always welfare improving for pure competition.

One natural extension is to consider the use of other financial instruments, particularly convertible bonds, as in Green (1984) and Haugen and Senbet (1983).[8] A convertible bond gives the bond holder the right to convert bonds into equity at a pre-specified transaction price up to some date. In our model the only meaningful conversion date is after uncertainty has been resolved but before final payoffs are made. This conversion option has value and would therefore reduce the premium

[6] Basic references on the theory of strategic commitment in industrial organization include Eaton and Lipsey (1980) and Dixit (1980). The symmetric structure described here would be formally similar to Brander and Spencer (1983).

[7] There is, however, also a limited liability effect of debt analyzed in Brander and Lewis (1986), that operates in the opposite direction, but that is absent in our model because of the assumption of "up-front" financing. In Brander and Lewis (1986), debt is taken on in stage 1, but production costs may be paid out of sales revenue or new equity in stage 2 as production occurs, and there is no effort decision. Increases in debt commit the firm to a riskier and more aggressive output strategy.

[8] Haugen and Senbet (1983) argue that, in the absence of limited liability, the use of options can eliminate moral hazard problems associated with "perquisite" consumption by owner-managers. Green (1984) concludes that using warrants or convertible debt as a response to the moral hazard associated with "excessive risk taking" under limited liability can reduce but not eliminate market failure.

that would be paid for debt, expanding the range of states over which the owner is the residual claimant, and ameliorating the moral hazard problem. However, in good states, conversion will take place and the owner will get less than the full residual returns. This reduces his incentive to provide effort, tending to worsen the moral hazard problem. An optimal convertible debt contract will normally dominate straight debt, but will not eliminate market failure, and the results of the paper continue to hold in slightly modified form.

University of British Columbia, Canada

APPENDIX

PROOF OF PROPOSITION 2. Parts i) and ii) have been proven in the text. As for part iii), substituting (3.2) and (3.7) into (3.9) and using $y_{Iz} = y_{ez} = 0$, we obtain

$$\text{(A1)} \quad dy/dI = \left[\int_{z^*}^{1} (y_I R_{ee} - y_e R_{eI}) f(z)\, dz - f(z^*) R_e^*(y_I z_e^* - y_e z_I^*) - G''(e) \right] \Big/ U_{ee}.$$

From (2.3) and (3.8), $y_I R_{ee} - y_e R_{eI} = (p + yp')(y_I y_{ee} - y_e y_{eI})$, which is negative from (2.4). From (2.3) and (2.7), $y_I z_e^* - y_e z_I^* = 0$. Since $G''(e) < 0$ and $U_{ee} < 0$, it follows that $dy/dI > 0$.

PROOF OF PROPOSITION 3. (i) Suppose the effort of a monopolist is set at the level that would be chosen by a competitive firm. Then from (2.3) and (3.1), $U_e(e, I, D) = \int_{z^*}^{1} yp' y_e(e, I, z) f(z)\, dz < 0$, indicating that the monopolist can increase utility by reducing effort below the competitive level (holding I and D constant). For a given B and E, from (2.7) and (2.9), both z^* and D increase as effort falls. Since $e_D < 0$ (from (2.6)), the result follows.

(ii) Suppose there are n identical competitive firms each with effort level e/n, investment level I/n and debt level D/n. If y is linearly homogeneous in e and I, then y_e is homogeneous of degree zero in e and I, and, from (2.2) and (2.5), z^* is homogeneous of degree zero in e, I, and D. Using these homogeneity conditions, from (3.1) the optimal choice of e/n by a competitive firm satisfies $U_{e/n} = \int_{z^*}^{1} p y_e(e, I, z) f(z)\, dz - nG'(e/n) = 0$. Since $G''(e) \geq 0$, the result (ii) follows on the basis of a similar argument to that used in 3(i).

PROOF OF PROPOSITION 4. i) From (4.4) and (4.6), $D_B - D_E = (1 + i)/a > 0$. Using (3.6) and (4.2), it follows that $e_B - e_E = e_D(1 + i)/a < 0$.

ii) From (4.7) $dy/dB - dy/dE = y_e(e_B - e_E) < 0$, using 4(i).

iii) From differentiation of (2.6) $dz^*/dB - dz^*/dE = z_e^*(e_B - e_E) + z_D^*(D_B - D_E) > 0$, from (2.7) and the proof of 4(i).

PROOF OF PROPOSITION 5. i) From (4.4), (4.6), (5.4) and (5.5), $\hat{U}_B = \hat{U}_E + v'(E) - (1 + i)(1 - F(z^*))/a$. Suppose now that $v'(E) = 1 + i$. Then, substituting for a from (4.4), we obtain

(A2) $$\hat{U}_B = \hat{U}_E + (1+i)\left[e_D \int_0^{z^*} R_e f(z)\,dz\right] \Big/ a < 0$$

using $e_D < 0$ and $a > 0$. It follows that $B = 0$ from (A2) and (5.4).

ii) At $B = 0$, we have $D = 0$ and $z^* = 0$. (There is no chance of bankruptcy). From (4.3) and (4.5), $D_B(0, E) = 1 + i$ and $D_E = 0$. Therefore, from (5.5) $\hat{U}_E(0, E^0) = \int_0^1 R_I f(z)dz - v'(E^0) = 0$, where E^0 is the maximizing level of E at $B = 0$. Also $\hat{U}_B(0, E^0) = \int_0^1 R_I f(z)dz - (1+i)$. Hence, if $v'(E^0) > (1+i)$, then $\hat{U}_B(0, E^0) > 0$ and the owner will choose $B > 0$.

Proof of Proposition 6. (i) Let the utility maximizing levels of effort and equity at $B = 0$ be $e^0 = e(I^0, 0)$ and E^0 respectively. (Note that $E^0 = I^0$.) Now suppose that the owner is given the option of borrowing. Then let B^1 and E^1 be the new utility maximizing levels of borrowing and equity, given $I = I^0$. Since $v'(E^0) > (1+i)$, we know that $B^1 > 0$ from Proposition 5(ii). The owner's effort is then $e = e(I^0, D^1)$, where $D^1 = D(B^1, E^1)$. Therefore, using the mean value theorem and (3.6), $e(I^0, D^1) - e(I^0, 0) = e_D D^1 < 0$ where e_D is evaluated at some intermediate level of (B, E) between $(0, E^0)$ and (B^1, E^1)). Similarly, $y(I^0, e(I^0, D^1), z) - y(I^0, e(I^0, 0), z) = y_e e_D D^1 < 0$.

ii) Holding investment fixed at I^0, (B^1, E^1) maximizes $\hat{U}(B, E)$ and satisfies $\hat{U}_E(B^1, E^1) = \hat{U}_B(B^1, E^1)$. Also, E^0 satisfies $\hat{U}_E(0, E^0) = 0$. From the mean value theorem, $B^1 = -(E^1 - E^0)$, and (6.1),

(A3) $$\hat{U}_E(B^1, E^1) - \hat{U}_E(0, E^0) = (\hat{U}_{EB} - \hat{U}_{EE})B^1 > 0.$$

Therefore, $\hat{U}_E = \hat{U}_B > 0$ at (B^1, E^1) and the owner will increase investment.

Proof of Proposition 7. (i) Let $\hat{W}(B, E, n) \equiv W(e(B, E), B, E, n)$ where $e = e(B, E)$ is given by (4.1) and W is total welfare as given in (3.3). From (3.3),

(A4) $$\hat{W}_B(B, E, n) = n\left[\int_0^1 p(dy/dB)f(z)\,dz - G'(e)e_B - (1+i)\right].$$

Substituting for $G'(e)$ from (3.1) in (A4) and using (4.7) and $R_e = py_e$ (from (2.3) for pure competition), we obtain $\hat{W}_B = n\hat{U}_B$ where $\hat{U}_B$ is given by (5.9). Similarly, from differentiation of (3.3) using (2.3), (3.1) and (4.7), and (5.10), $\hat{W}_E = n\hat{U}_e$. Also, from differentiation of (3.3) with respect to n using (2.8) and (2.9), $\hat{W}_n = U(B, E)$. Hence utility maximization and free entry (under pure competition) ensures that $\hat{W}_B = 0$, $\hat{W}_E = 0$ and $\hat{W}_n = 0$. These are the first order conditions for the (second best) welfare maximizing choices of B, E, and n. The second order conditions are implied by (5.6).

(ii) From Proposition 5, if $v'(E^0) > (1+i)$ at $B = 0$, the owner will choose positive amounts of debt. By revealed preference the owner's utility must have increased. Under pure competition, the proof of 7(i) above implies that, for any fixed number of firms, welfare increases as long as the utility of the owner increases. Free entry ensures that the second best efficient level of entry is achieved.

REFERENCES

ALLEN, F., "Capital Structure and Imperfect Competition in Product Markets," discussion paper, University of Pennsylvania, 1985.

BENOIT, J. P., "Financially Constrained Entry in a Game with Incomplete Information," *Rand Journal of Economics* 15 (1984), 490–499.

BRANDER, J. A. AND T. R. LEWIS, "Oligopoly and Financial Structure: The limited liability effect," *American Economic Review* 76 (1986), 956–970.

——— AND ———, "Bankruptcy Costs and the Theory of Oligopoly," *Canadian Journal of Economics* 21 (1988), 211–243.

——— AND B. J. SPENCER, "Strategic Commitment with R & D: The Symmetric Case," *Bell Journal of Economics* 14 (1983), 225–335.

DARROUGH, M. AND N. STOUGHTON, "Moral Hazard and Adverse Selection: The Question of Financial Structure," *Journal of Finance* 41 (1986), 501–513.

DIXIT, A., "The Role of Investment in Entry Deterrence," *Economic Journal* 90 (1980), 95–106.

DYE, R., "Optimal Monitoring Policies in Agencies," *The Rand Journal of Economics* 17 (1986), 339–350.

EATON, B. C. AND R. G. LIPSEY, "The Durability of Capital as a Barrier to Entry," *Bell Journal of Economics* 11 (1980), 721–729.

FAMA, E., "Agency Problems and the Theory of the Firm," *Journal of Political Economy* 88 (1980), 288–307.

GREEN, R. C., "Investment Incentives, Debt, and Warrants," *Journal of Financial Economics* 13 (1984), 115–136.

HAUGEN, R. AND L. SENBET, "Resolving the Agency Problems of External Capital Through Options," *Journal of Finance* 36 (1983), 629–647.

HICKS, J., "Annual Survey of Economic Theory: The Theory of Monopoly," *Econometrica* 3 (1935), 1–20.

HOLMSTROM, B., "Moral Hazard and Observability," *Bell Journal of Economics* 10 (1979), 74–91.

JENSEN, M. AND W. MECKLING, "Theory of the Firm: Managerial Behaviour, Agency Costs, and Capital Structure," *Journal of Financial Economics* 3 (1976), 305–360.

JOHN, K. AND D. NACHMAN, "Risky Debt, Investment Incentives, and Reputation in a Sequential Equilibrium," *Journal of Finance* 40 (1985), 863–878.

KRAUS, A. AND R. H. LITZENBERGER, "A State-Preference Model of Optimal Financial Leverage," *Journal of Finance* 33 (1973), 911–922.

LELAND, H. AND D. PYLE, "Information Asymmetries, Financial Structure and Financial Intermediation," *Journal of Finance* 32 (1977), 371–387.

MAKSIMOVIC, V., "Optimal Capital Structure in Oligopolies," Ph.D. Dissertation, Harvard University, 1986.

MODIGLIANI, F. AND M. MILLER, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48 (1958), 261–297.

MYERS, S. C., "Determinants of Corporate Borrowing," *Journal of Financial Economics* 4 (1977), 147–175.

PAULY, M., "Overinsurance and the Public Provision of Insurance: the Roles of Moral Hazard and Adverse Selection," *Quarterly Journal of Economics* 88 (1974), 44–54.

POITEVIN, M., "Three Essays on the Strategic Interaction Between Production and Financial Decisions," Ph.D. Dissertation, University of British Columbia, 1988.

RAVID, S. A., "On the Interactions of Production and Financial Decisions," *Financial Management* 17 (1988), 87–99.

ROSS, S., "The Determinants of Financial Structure: The Incentive-Signalling Approach," *Bell Journal of Economics* 8 (1977), 23–40.

SAPPINGTON, D., "Limited Liability Contracts Between Principal and Agent," *Journal of Economic Theory* 29 (1983), 1–21.

SHAVELL, S., "Risk Sharing and Incentives in the Principal and Agent Relationship," *Bell Journal of Economics* 10 (1977), 74–91.

SMITH, A., *An Enquiry into the Nature and Causes of the Wealth of Nations*, 2nd Edition (Chicago: University of Chicago Press, 1976).

STIGLITZ, J. E., "Incentives and Risk Sharing in Sharecropping," *Review of Economic Studies* 41 (1974), 219–255.

TITMAN, S., "The Effect of Capital Structure on a Firm's Liquidation Decision," *Journal of Financial Economics* 13 (1984), 137–151.

WILLIAMS, J., "Perquisites, Risk, and Capital Structure," *Journal of Finance* 42 (1987), 29–49.

WILLIAMSON, O. E., *Markets and Hierarchies: Analysis and Anti-trust Implications* (New York: Free Press, 1975).