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SECOND BEST PRICING OF PUBLICLY PRODUCED INPUTS

The case of downstream imperfect competition

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Efficient second best pricing is examined for a public enterprise facing two distortions: a profit constraint and imperfect competition among some customer industries. We suggest a measure of downstream industry distortion for the purpose of efficient pricing. The pricing rule contains two opposing elements: the shadow value of public profit and this measure of the downstream distortion, whose sum determines whether the efficient second best input price is above or below marginal cost. Efficient pricing normally implies relative subsidization of imperfectly competitive downstream firms.

1. Introduction

Almost all of the optimal pricing literature has assumed that the output of public enterprises is for final consumption. In practice, of course, public enterprises also produce outputs that are used by private firms as inputs to further production. In a 'first best' environment, without profit constraints, redistributional objectives, or distortions in other markets, the optimality of marginal cost pricing is not affected. Feldstein (1972) considers optimal pricing of publicly produced inputs purchased by competitive downstream industries. In his paper, departures from marginal cost pricing arise from a minimum profit constraint on the public enterprise and from distributional objectives. In this paper the focus is on departures from marginal cost pricing induced by imperfect competition in the industries which purchase publicly produced inputs. We abstract from distributional concerns but do consider the effects of a profit constraint. The analysis is most closely related to the branch of the 'second best' literature that focuses on distortions in the rest of the economy as a reason for departures from marginal cost pricing.

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[see, especially, Lipsey and Lancaster (1956), Davis and Whinston (1967) and Farrell (1958).]

The basic insight of this second best literature for optimal public pricing is stated in Atkinson and Stiglitz (1980, p. 468): 'If private firms price above marginal cost, and if their output is an increasing function of the public price, then we shall on this account want to raise the price above marginal cost.' (See also Turvey (1971).)

In much of the work on the second best the relationship between publicly produced goods and goods produced under imperfect competition arises because of cross effects of demand. If, for example, the publicly produced good is a substitute for an underproduced good its efficient price would be higher than otherwise. However, the statement in Atkinson and Stiglitz applies equally well to the case under consideration here. Provided that the publicly produced input is a normal factor,[1] reductions in its price will induce socially desirable increases in the outputs of imperfectly competitive firms. In the absence of other distortions, such prices should be below marginal cost.[2] A measure, denoted σ, is derived which shows the magnitude of the downstream distortion for the purposes of efficient pricing. The measure σ enters the efficient pricing equation symmetrically with the shadow value of public profit.

2. The model

We are concerned with pure efficiency objectives. Accordingly, it is assumed that the marginal utility of income is constant (and normalized to equal unity for each consumer) for the changes under consideration here, and that consumer preferences and welfare can be represented by a community indirect utility function. Since the focus is on efficient pricing adjustments induced through the production relationship, we abstract from demand interdependence.

The public enterprise produces an output of which quantity X^* is purchased by final consumers at price r^*, and quantity X is purchased by downstream firms at price r. Implicitly there are many downstream industries, each being charged a different input price. For ease of notation, however, only a single downstream industry is modelled. Generalization

[1]Following Ferguson (1969) a normal factor is defined as one which induces an increase in the downstream firm's output as its price falls.

[2]The desirability of this input-distorting subsidy contrasts with the well-known result of Diamond and Mirrlees (1971) on optimal tax theory that inputs should not be taxed or subsidized. One obvious source of this difference is that our partial equilibrium model reflects an institutional setting in which a particular regulatory authority has control over only one or a few prices and, in particular, cannot tax or subsidize outputs of downstream firms. Even in a general equilibrium context, unless a social planner has full taxing power over all commodities and factors, distortionary factor taxes may be efficient in some industries. [Atkinson and Stiglitz (1980, ch. 15).

follows immediately upon adding superscripts to identify different industries. The representative downstream industry has n identical firms and produces output Y for sale at price p. Each firm has output y, input use x and profit π.

The profit, B, of the public enterprise is

$$B(r^*, r) = r^* X^* + rX - K(X + X^*), \tag{1}$$

where K is the total cost function (and k is marginal cost). The model is a partial equilibrium model embracing two final goods markets: the market for publicly produced good X^* and the market for the downstream good Y. Accordingly, the indirect utility function V is written as a function of consumer price r^*, the price p of the downstream good which in turn depends the input price r, and of the contribution of profit from the two markets to income:

$$V = V(r^*, p(Y(r)), n\pi(r) + B(r^*, r)). \tag{2}$$

Other prices and other income are not explicitly written as arguments of the indirect utility function since they are assumed external to the current problem.

One important aspect of public pricing is that a public enterprise may be constrained to meet a minimum profit target. [Efficient pricing in the presence of such a constraint is often referred to as Ramsey pricing. References include Ramsey (1927), Boiteux (1956), Baumol and Bradford (1970), and Hartwick (1978).] We allow for a profit target $\bar{B}$ (which may be negative). In this case the problem of the regulator is to maximize V subject to the profit constraint, using r^* and r as choice variables. The associated Lagrangian function is

$$L = V + \mu[B(r^*, r) - \bar{B}], \tag{3}$$

where μ is the Lagrange multiplier associated with the profit constraint. This constrained maximization problem coincides with the unconstrained maximization of V if $\mu = 0$. Using p' to denote the (strictly negative) derivative of inverse demand with respect to Y and subscripts to denote other derivatives, the first-order conditions associated with (3) are as follows:

$$L_r^* = V_r^* + (V_B + \mu)B_r^* = 0, \tag{4}$$

$$L_r = V_p p' Y_r + nV_\pi \pi_r + (V_B + \mu)B_r = 0. \tag{5}$$

The marginal utility of income has been normalized to equal 1. Therefore

$V_\pi = V_B = 1$. We assume that consumers maximize utility so Roy's Identity[3] yields $V_r^* = -X^*$ and $V_p = -Y$. Differentiating (1) with respect to r^* yields $B_r^* = X^* + r^* X_r^* - kX_r^*$ so that (4) becomes

$$(1+\mu)(r^*-k)X_r^* + \mu X^* = 0, \tag{6}$$

which is the normal result for final consumers. Eq. (5) can be rewritten:

$$-Yp'Y_r + n\pi_r + (1+\mu)B_r = 0. \tag{7}$$

To simplify (7) note that

$$B_r = (r-k)X_r + X, \tag{8}$$

and that the profit of a downstream firm is given by $\pi = yp(Y) - c(y;r)$, where c is its cost function. Assuming cost minimization, from the envelope theorem $\partial c/\partial r = x$, there

$$\pi_r = (p-c')y_r + yp'Y_r - x, \tag{9}$$

where c' denotes marginal cost. If we let

$$\sigma = (p-c')Y_r/X, \tag{10}$$

and then insert (8), (9), and (10) into (7), we obtain:

$$(1+\mu)(r-k)X_r + (\mu+\sigma)X = 0. \tag{11}$$

Expressions (6) and (11) can be written in the more familiar elasticity forms:

$$\frac{(r^*-k)}{r^*} = \frac{\mu}{(1+\mu)} \frac{1}{\eta_r^*}, \tag{12}$$

$$\frac{(r-k)}{r} = \frac{(\mu+\sigma)}{(1+\mu)} \frac{1}{\eta_r}, \tag{13}$$

where $\eta_r^* = -X_r^* r^*/X^*$ and $\eta_r = -X_r r/X$.

Expressions (10) and (13) require no assumptions about the behaviour of downstream firms apart from cost minimization and that the input demand

[3]A reference for Roy's Identity, the envelope theorem, and other duality results, is Varian (1978).

function $X(r)$, output function $Y(r)$, and their derivatives are well defined. Expressions (12) and (13) are the markup equations for second best pricing. The only structural difference between them is the presence of the variable σ in the input markup equation. Furthermore, if the downstream industry were perfectly competitive so that $p=c'$, σ would equal zero. This implies that Ramsey pricing is structurally identical for final consumers and for perfectly competitive downstream industries, although the actual Ramsey prices would differ if the elasticities η_r^* and η_r differed.

Following Ferguson (1969), X is defined as a normal factor[4] if an increase in its price lowers output in the downstream industry, that is, if $Y_r<0$. Therefore, under imperfect competition (with $p>c'$), σ will normally be negative and will be greater in absolute value as the divergence between price and marginal cost is greater.

For the unconstrained case, μ is zero and σ is the only source of departure from marginal cost pricing. Eq. (13) is then $(r-k)/r=\sigma/\eta_r$. In the normal case ($\sigma<0$), the required departure will be negative: imperfectly competitive industries would be charged less than marginal cost.

Proposition 1. With $\mu=0$:

(a) *price equals marginal cost ($r=r^*=k$) for final consumers and competitive downstream firms*, and

(b) *price is lower than marginal cost ($r<k$) for imperfectly competitive firms if X is a normal factor, and price exceeds marginal cost if X is inferior.*

Expression (13) suggests that the relevant measure of distortion for second best pricing adjustments is the variable σ, which is $(p-c')Y_r/X$. σ contains the three elements that are important for the pricing adjustment: the excess price, $p-c'$; the responsiveness of downstream output to input price, Y_r; and input use X. If $p-c'$ is small, there are only small gains to be made by increasing downstream output. If Y_r is small, input price adjustments have very little effect on downstream output and in the limit, with $Y_r=0$, there is no efficiency gain whatsoever from pricing adjustments. X appears in the denominator of σ because, for any given elasticity of input use, a high level of X is associated with a high value of X_r and a large input mix distortion for each unit change in r.

Proposition 2. ($\mu\geqq 0$) *Let σ_i and σ_j be the values of σ evaluated at the Ramsey optimum for two industries with the same values of η_r. Then $\sigma_i<\sigma_j$ implies $r_i<r_j$.*

[4]An inferior factor is one for which $Y_r>0$. An alternative definition is that expansions in output lower input demand: $X_Y<0$. These definitions coincide for most reasonable models. (For example, under downstream monopoly $Y_r=X_Y/\pi_{YY}$, and $\pi_{YY}<0$ is the second-order condition.) However, a factor cannot be a Giffen factor: X_r must be negative and η_r must be positive [see Ferguson (1969)].

Proposition 2 includes both the normal factor and inferior factor possibilities. In the normal case, a more negative σ indicates greater distortion and induces a larger relative subsidy. In the case of an inferior factor a more positive σ indicates greater distortion and induces a larger relative markup. If $\mu > 0$, imperfectly competitive firms would (normally) be subsidized relative to competitive firms, but might still be charged an input price above marginal cost.

Our analysis is easily extended to handle firms of different sizes and with different cost curves within the same industry. Replacing $n\pi$ with $\sum \pi^i$ and $n\pi_r$ with $\sum \pi_r^i$ (where i is an index over firms) in the objective function (2) and the first-order condition (5), respectively, yields a slightly generalized measure of downstream distortion, $\sigma = \sum_i (p - c_i')(\partial y_i/\partial r)/X$ with which to set the uniform industry input price. The measure σ is then a weighted average of the price cost margins in the individual firms. Nothing else is changed.

3. Extensions and concluding remarks

We examine second best pricing in the presence of two distortions: the need for a public enterprise to meet a (possibly negative) profit target, and imperfect competition in downstream industries that purchase the output of the public enterprise. For each industry there is only one instrument, the price of the publicly produced input, to deal with the two distortions. A profit target tends to induce an input price above marginal cost; downstream imperfect competition normally calls for an input price below marginal cost so as to induce socially desirable additions to output in the downstream industry. Consequently, the markup equation contains two offsetting elements, the shadow value of public profit, μ, and a variable, labelled σ, which captures, for any particular elasticity of input demand, the relevant aspects of the downstream distortion. The resulting second best input price is above or below marginal cost according to whether $\mu + \sigma$ is positive or negative.

As pointed out by a referee, in practice, prices of industrial inputs provided by public enterprises may vary across firms of different sizes rather than across industries. Moreover, if large users tend to negotiate contracts individually, the assumption that the public enterprise can simply set prices seems less appropriate. This latter point raises some interesting second best problems but is beyond the scope of the present analysis.

However, our framework can be modifed to handle firm-specific input prices (set by the public enterprise) for the general case of differentiated products with demand interdependence. The indirect utility function becomes

$$V = V(r^*, \tilde{p}(\tilde{y}(\tilde{r})), \ \sum_j \pi^j(\tilde{r}) + B(r^*, \tilde{r})),$$

where tildes denote vectors. The measure of firm-specific distortion is then

$$\sigma^i = \sum_j (p_j - c_j')(\partial y_j/\partial r^i)/x_i.$$

If products i and j are substitutes, $\partial y_j/\partial r_i$ is normally positive which tends to make σ^i less negative and raise the efficient price r_i. The pricing rule becomes:

$$(r_i - k)/r_i = [(\mu + \sigma^i)/(1+\mu)](x_i/X\eta_r^i),$$

where $\eta_r^i = -(r_i/X)\partial X/\partial r_i$ which differs in form from our previous rule (13) only by the factor x_i/X.

The result that imperfectly competitive firms should be subsidized seems, like the standard Ramsey prescription, unpalatable on distributional grounds. Naturally, explicit incorporation of distributional judgements would alter the structure of optimal input prices. It seems worth recognizing, however, that failure to take account of imperfect competition does involve a departure from efficiency.

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