

1 Review of vectors and matrices

Following our notation in class, let

$$a = 1$$

$$b = 2$$

$$c = 50$$

$$d = 63$$

1. Let vector $\vec{r} = [a, b]$. Write out \vec{r} in numbers. $[1, 2]$
2. Write out \vec{r}' . $[\frac{1}{2}]$
3. Write out $\vec{r} + \vec{r}$. $[1, 2] + [1, 2] = [2, 4]$
4. Let vector $\vec{v} = [c, d]$. Write out \vec{v} . $[50, 63]$
5. Write out $\vec{v} + \vec{r}$. $[50, 63] + [1, 2] = [51, 65]$
6. Write out $\vec{v} + \vec{r}'$. $[50, 63] + [\frac{1}{2}]$
7. $A = [\vec{r}', \vec{v}']$ is a 2×2 matrix. Write it out. $A = \begin{bmatrix} \frac{1}{2} & 50 \\ 2 & 63 \end{bmatrix}$
8. Write out A' . $A' = \begin{bmatrix} 1 & 2 \\ 50 & 63 \end{bmatrix}$
9. If we write $2 \times \vec{v}$, that means you should multiply each element in \vec{v} by 2. Write out $2 \times \vec{v}$. $2 \times [50, 63] = [100, 126]$
10. Write out $2 \times A$. $2 \times \begin{bmatrix} \frac{1}{2} & 50 \\ 2 & 63 \end{bmatrix} = \begin{bmatrix} 1 & 100 \\ 4 & 126 \end{bmatrix}$
11. Write out the set $\tilde{S} = \{A, [\vec{r}, \vec{v}]\}$. $\tilde{S} = \left\{ \begin{bmatrix} \frac{1}{2} & 50 \\ 2 & 63 \end{bmatrix}, [(1, 2), (50, 63)] \right\}$
12. Write out the set $\tilde{S} = \{A, A'\}$. $\tilde{S} = \left\{ \begin{bmatrix} \frac{1}{2} & 50 \\ 2 & 63 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 50 & 63 \end{bmatrix} \right\}$
13. Write the matrix $B = \begin{bmatrix} 2 & 4 \\ 150 & 189 \end{bmatrix}$ in terms of \vec{v} and \vec{r} . $B = \begin{bmatrix} 2 \times \vec{r} \\ 3 \times \vec{v} \end{bmatrix}$
14. If we write $A(i, j)$, it means we are referring to a specific element in A . The first argument (i) refers to the row in A that we want to look at, and the second argument (j) refers to the column we want to look at. So if we write $A(1, 2)$, it means we want to look at the number in the first row and in the second column (think of " Battleship"). We always start counting rows and columns from the upper left corner of the matrix. What is $A(1, 2)$? $A(1, 2) = 50$
15. What is $A(2, 1)$? $A(2, 1) = 2$
16. If we replaced $A(1, 2)$ with a "5", what would A look like?

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 63 \end{bmatrix}$$

2 Coordinate systems

In class we saw that if we took a fixed set of vectors and plotted them in different coordinate systems, the resulting object that we plotted changed dramatically. Now we're going to take a fixed object and try to describe it in different coordinate systems. Changing coordinate systems changes the description, but not the structure of the object (so long as we do it right). **Print out** the next several pages to complete this section.

On the next page (in Figure 1) you'll see that an object that occupies a portion of the sheet of paper. Because this object is defined by a set of six corners, we'll call it the set \tilde{G} (perhaps it's a bad map of MacMillan). Your job is to characterize \tilde{G} using five different coordinate systems, shown on the following pages. Each coordinate system gives a numerical language for describing specific locations on the page. To make sure that you properly map \tilde{G} onto each coordinate system, **take the printout of each coordinate system and place it on top of the printout of \tilde{G}** (making sure the corners line up); then trace \tilde{G} .

Each coordinate system has two dimensions, A and B . For simplicity, let's always list the location in the A dimension first and the location in the B dimension second (it could be the other way around and it would be okay, so long as we are clear about which convention we use). So we'll define the object \tilde{G} by listing the set of corners $\{[A_1, B_1], [A_2, B_2], \dots, [A_6, B_6]\}$.

1. Create a table like the following one to list the corners of \tilde{G} using the various coordinate systems depicted on the following pages. Make sure that both the location and shape of the object are preserved when you list the corners in each coordinate system. Use 1 decimal point of accuracy (we will allow for ± 0.5 in measurement error when grading).

Coord. System	1	2	3	4	5	6
"Cartesian XY"	(3.1, 3.2)	(3.1, 6.8)	(4.5, 6.8)	(4.5, 5.9)	(7.1, 5.9)	(7.1, 3.2)
"Cartesian IJ"	(7.1, 3.2)	(3.2, 3.1)	(3.2, 4.5)	(4.1, 4.5)	(4.1, 7.1)	(7.1, 7.1)
"Skew"	(6.7, 2.4)	(8.5, 2.5)	(6.9, 7.1)	(6.2, 5.9)	(3.2, 4.9)	(0.5, 1)
"Polar rectangular"	(4, 6.2)	(2.9, 7.7)	(2.8, 0.3)	(1.3, 7.8)	(1.8, 2.5)	(3.9, 4)
"Seuss"	(1.2, 2.2)	(3.2, 1)	(3.9, 1.5)	(2.9, 1.5)	(2.5, 3.5)	(2.5, 4.7)

2. Do you prefer a specific system? Why? *Cartesian XY because it is the system that is the easiest to follow with x-axis on bottom and y-axis on the side.*
3. Sometimes we document the location of objects using one coordinate system, but then try to reconstruct them using a different coordinate system.
 - (a) Taking your description of \tilde{G} using the "Cartesian IJ" system (the set of vectors you wrote down under that column in the table above), try to reconstruct the object by plotting these points out in the "Cartesian XY" system (you'll probably need to print an extra sheet of Figure 2 to do this so you can turn in your drawing). Does \tilde{G} look right? In which ways does it look wrong? *No, \tilde{G} is turned 90° to the left.*
 - (b) Taking your description of \tilde{G} using the "Skew" system, try to reconstruct the object by plotting these points out in the "Cartesian XY" system. Does \tilde{G} look right? In which ways does it look wrong? *No, \tilde{G} is flipped along y-axis and stretched.*
 - (c) Taking your description of \tilde{G} using the "Seuss" system, try to reconstruct the object by plotting these points out in the "Cartesian XY" system. Does \tilde{G} look right? In which ways does it look wrong? *No, \tilde{G} is flipped along the x-axis, turned 35° to the left and tilted upward.*

Collaborators

Please list everyone you worked on this assignment with outside of public class Discussion posts.

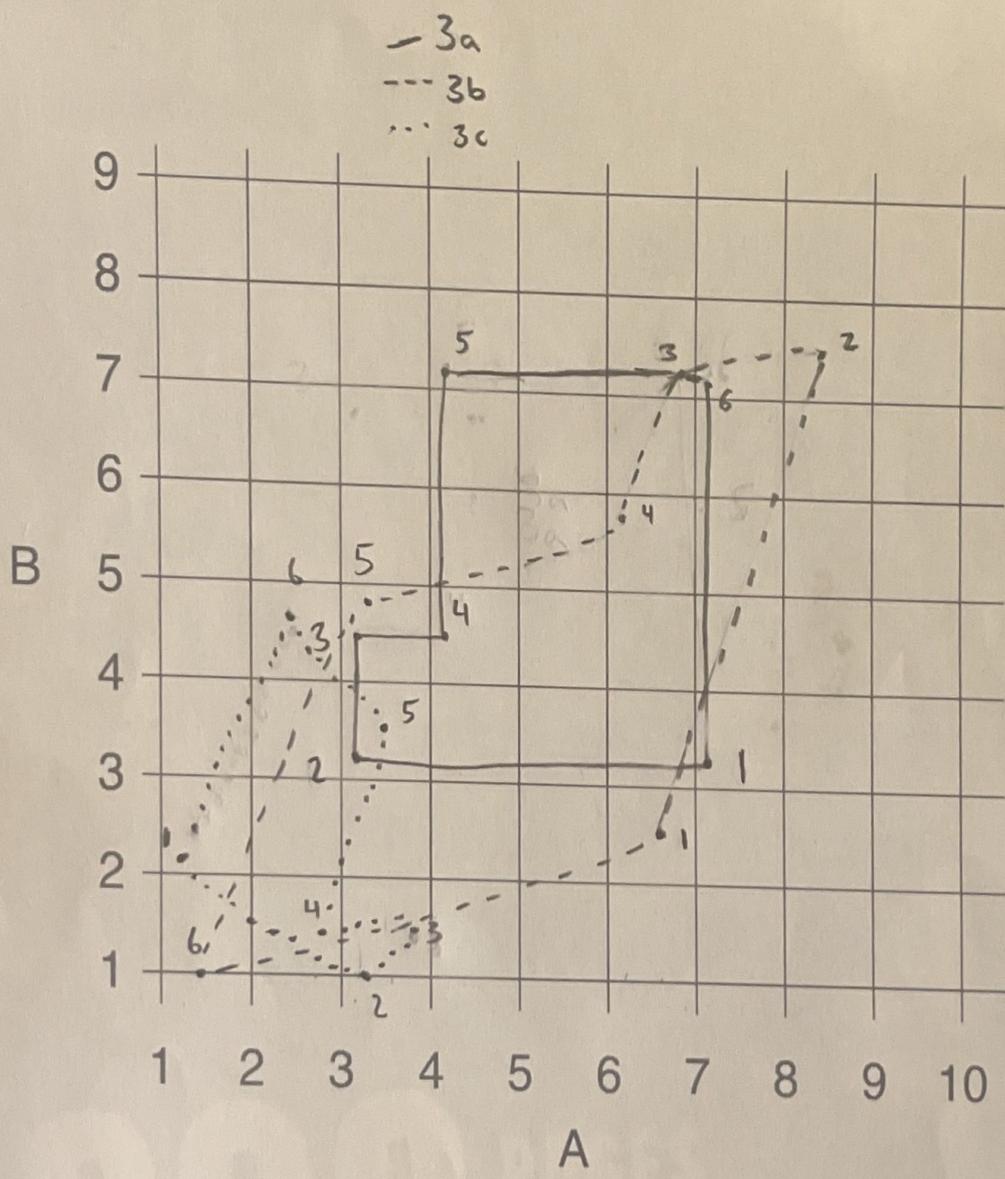


Figure 2: Coordinate system "Cartesian XY"