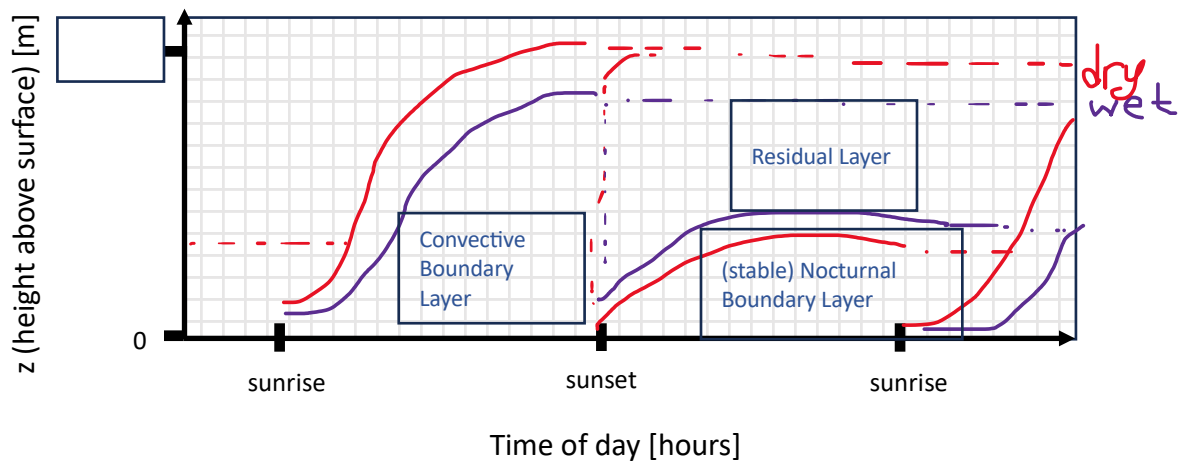


HW4 draft

-2 points from total from 7(g) – skip/don't bother marking that one!

1. [8]

Draw the day-night evolution of the depth of the atmospheric boundary layer over a typical land surface (e.g. a grassy field) [2], and fill in the box on the upper end of the y-axis [1] (approximate value is fine); label the convective boundary layer, the residual layer, and the nocturnal boundary layer [3]. Using a different colour (or different type of line), draw how the boundary layer would look different if the surface were much wetter (e.g. a bog) [2].



Box label: ~2000 m (anything from 1000-3000 m is fine)

Daytime boundary layer should start small and grow with hours of daylight. Night boundary layer should start small and grow not as big as the day boundary layer. Residual layer is the “left over” day boundary layer from the previous day sitting above the “active” nocturnal boundary layer.

Over wet surfaces, we expect daytime boundary layers to be smaller (less surface heating), but nighttime boundary layers to be larger (warmer surface as wet surfaces cool less than dry surfaces at night).

Rubric:

[2] for approximately correct day/night heights

[3] 1 per label of convective, residual, and nocturnal boundary layers

[2] for approximately correct day/night heights over wetter surface

2. [6]

Use the ideal gas law to calculate the density ρ_1 and ρ_2 of two parcels of air, both at

1000 hPa (sea level pressure) [4]. The temperature T_1 of the first air parcel is 10°C , and the temperature T_2 of the second is 15°C . Which parcel would rise above the other, and why [2]?

Starting with the ideal gas law:

$$\begin{aligned} pV &= mR_sT \\ p &= m/V R_sT \\ p &= \rho R_sT \\ \rho &= \frac{p}{R_sT} \end{aligned}$$

For dry air, the specific gas constant $R_s = 287 \text{ J/kg/K}$. We are told the pressure $p = 1000 \text{ hPa}$, which is $100,000 \text{ Pa}$; one Pascal is equal to 1 kg/m/s^2 . We are told temperatures are 10 and 15 C , which we have to add 273.15 to to convert to K .

$$\begin{aligned} \rho_1 &= \frac{p}{R_sT} \\ \rho_1 &= \frac{100000 \text{ kg/m/s}^2}{287 \text{ J/kg/K} \times (273.15 + 10) \text{ K}} \\ \rho_1 &= \frac{100000 \text{ kg/m/s}^2}{287 \text{ kg m}^2/\text{s}^2/\text{kg/K} \times (273.15 + 10) \text{ K}} \\ \rho_1 &= \frac{100000 \text{ kg}}{287 \times (273.15 + 10) \text{ m}^3} \\ \rho_1 &= 1.27544 \text{ kg/m}^3 \end{aligned}$$

Similarly for T_2 ,

$$\begin{aligned} \rho_2 &= \frac{10000 \text{ kg}}{287 \times (273.15 + 15) \text{ m}^3} \\ \rho_2 &= 1.27536 \text{ kg/m}^3 \end{aligned}$$

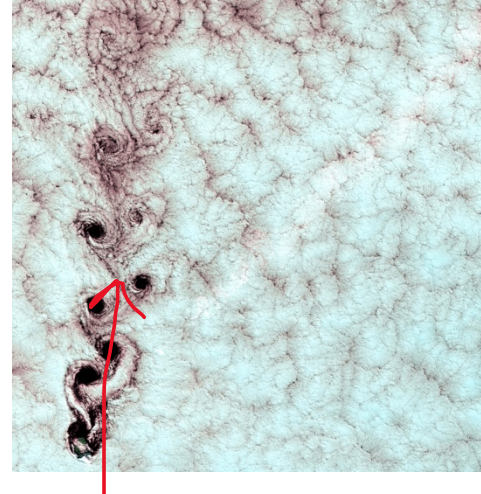
Because parcel 2 is warmer, it is less dense, thus it would rise above parcel 1.

Rubric:

- [2] for approach/ideal gas law implementation
- [2] for correct densities (hint: watch your units)
- [2] for correct answer + reason

3. [8]

The following satellite image was captured by Landsat 7 on September 15, 1999, showing von Karman vortices near Alexander Selkirk Island in the South Pacific. Using concepts covered in lecture:



- a. Indicate direction of mean wind (draw an arrow on the image). [1]



- b. Briefly describe the conditions that lead to the formation of these von Karman vortices. Are they always present in this location? [4]

[2] for some of:

To get von Karman vortices, you need (a) an obstacle, in this case the island, and (b) some background flow (horizontal wind). [1] The wind needs to be going fast enough to break into turbulence downwind of the obstacle – if it is going too slowly, no vortices are produced [1]. We can see the vortices here because of the clouds, but they can occur with or without cloud cover – we just can't see them [1].

[2] for:

They are not always present [1], and only occur when conditions support flow around the island that is strong enough to produce turbulence but not so strong to have the flow go entirely over the mountain. [1]

- c. Write the equation for the Froude number. Estimate the approximate value of the Froude number based on the flow visible in the image. [3]

The Froude number measures the magnitude of the horizontal winds vs. the stability of the atmosphere and the size of the obstacle (in this case, an island).

$$Fr = \frac{\pi U_0}{NH} \quad [1] \text{ for equation}$$

where U_0 is the mean horizontal wind speed, N is the Brunt-Vaisala Frequency which measures the vertical stability of the atmosphere, and H is the height of the obstacle. [1] for defining terms in equation

We expect the Froude number to be small and positive for this image, reflecting wind going around the sides of the island (then colliding and producing

horizontal turbulence). Froude numbers up to ~0.5 are acceptable. [1] for plausible value from 0-0.5. (As long as it is smaller than 1 and they have the correct reasoning, give the point).

We don't expect a large positive Froude number as that would mean wind going up and over the island, so you wouldn't get the splitting flow around the sides colliding to create the von Karman vortices. Additionally, you'd expect characteristics of downwind vertical waves like roll clouds (would look like stripes of clouds from above) if the Froude number was large.

Rubric:

a – [1] for correct direction

b – [3] for conditions, [1] for if they're always present or not

c – [1] for equation, [2] for approximate value estimate

4. [6]

In the process of photosynthesis, energy is extracted from photons in the PAR range. To assimilate one mole of CO₂, it requires an energy of 469 kJ. The same amount is released back during respiration (metabolism, decomposition of organic matter). We call this energy flux density the net biochemical energy storage ΔQ_p . Consider a location where at noon, net radiation Q^* is 600 W/m². A flux tower measures net vertical exchange of CO₂ to be -4.01 $\mu\text{mol}/\text{m}^2/\text{s}$ (i.e. 4.01 $\mu\text{mol}/\text{m}^2/\text{s}$ into the surface); this reflects plant uptake of CO₂ through photosynthesis.

a. [4] Calculate ΔQ_p

$$\begin{aligned}\Delta Q_p &= (469 \text{ kJ} / \text{mol}) \times (1000 \text{ J} / \text{kJ}) \times -4.01 (\mu\text{mol} / \text{m}^2 / \text{s}) \times (1 \text{ mol} / 10^6 \mu\text{mol}) \\ &= (469 \times 1000 \times -4.01 \times 10^{-6}) \text{ J}/\text{m}^2/\text{s} \\ &= -1.88 \text{ W}/\text{m}^2\end{aligned}$$

[2] for approach, [1] for answer, [1] for units.

The negative just means into the surface, i.e. the surface is taking up energy by fixing CO₂ (net downwards flux of CO₂ means photosynthesis > respiration).

b. [2] What fraction of Q^* is ΔQ_p ? Is this a substantial part of the surface energy budget? Discuss your answer.

$$\frac{\Delta Q_p}{Q^*} = \frac{-1.88}{600} = -0.003 \quad [1]$$

The magnitude of ΔQ_p is only 0.3% of the magnitude of net radiation Q^* . That is, the energy associated with the uptake of carbon via photosynthesis is only a very small contributor to the surface energy budget. [1]

NOT REQUIRED:

Even though ΔQ_p is only a small portion of the surface energy budget, photosynthetic uptake of CO₂ from the atmosphere is a huge component of the global energy budget, as land plants (let alone ocean biological uptake) remove roughly 30% of anthropogenic CO₂ emissions. CO₂ is a strong greenhouse gas. Without this land uptake of CO₂, the planet would be warming at a higher rate than it already is.

Rubric:

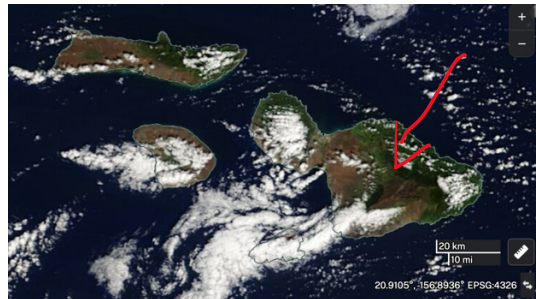
a – [2] approach, [1] numerical answer, [1] units

b – [1] numerical answer, [1] discussion.

5. [3] Each of the following examples has a sharp gradient in topography. Based on distribution of vegetation, say something about the direction of the mean winds.

If anyone talks about anabatic or katabatic winds, great! But not required!

a. Hawaii:



Greener vegetation reflects the higher precipitation amounts on the wind-ward side of the islands. These are the Trade Winds, which climatologically flow from the north east to the south west over Hawaii.

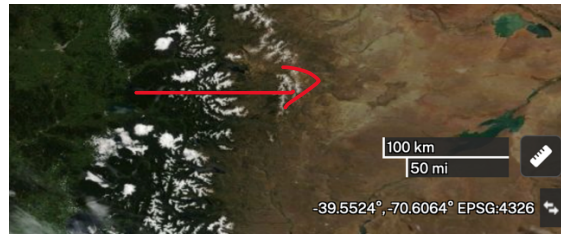
b. South America:



Greener vegetation reflects the higher precipitation amounts on the east side of

the Andes at this latitude; to the west is a high altitude desert.

c. South America:



In mid-latitude South America, the dominant direction of the winds switches to westerlies (vs b that has easterlies). This is similar to mid-latitude North America. Now the heavy precipitation is on the west side of the Andes.

6. [12]

What shape would you expect flow for each of the following Richardson numbers, and why? Are there eddies? Describe/discuss what is physically driving the motion in each case.

For the following answers, we use the fact that the Richardson number compares the magnitude of the thermal suppression or production of turbulence (numerator) to the mechanical production of turbulence (denominator). When thermal suppression is big, the Richardson number is large and positive, conditions are stable: turbulence is suppressed, and fluid motions occur in waves. When mechanical production becomes large, with moderate thermal suppression or production of turbulence, conditions become neutral; these are Richardson numbers around zero. There are isentropic eddies that are comparable in size in the vertical and horizontal directions. Small positive Richardson numbers correspond to pancake shaped eddies (thermal suppression and mechanical production). Large negative Richardson numbers correspond to large thermal production of turbulence. These are unstable conditions, and eddies are larger in their vertical extent than in their horizontal extent.

a. $R_f = 50$



This flow would not have turbulence. Flow would occur in waves, without turbulent eddies. [1]

The thermal suppression of turbulence would be much larger than the

mechanical production of turbulence (and there would be thermal suppression, rather than production, of turbulence). [1]

b. $R_f = 0.7$

 [1]


There would be stably stratified turbulence [1]. Eddies would be pancake-shaped. There would be thermal suppression of turbulence and mechanical production of turbulence. [1]

c. $R_f = -0.5$

 [1]

Flow is unstable, but only a bit more unstable than neutral conditions. [1] There is both mechanical and thermal production of turbulence [1]. Eddies would be taller than they are wide, but only by a bit; they aren't isentropic (roughly symmetrical), but nor are they extremely tall compared to their width.

d. $R_f = -3$

 [1]

Flow is unstable [1], dominated by thermal production of turbulence [1], possibly with some mechanical production of turbulence. Eddies are substantially larger in the vertical dimension than the horizontal direction.

Rubric:

[1] per shape, [2] per reasoning

7. [12]

The August-Roche-Magnus equation for calculating the saturation vapour pressure p (in kilopascals, or kPa) of water in air as a function of temperature T (in degrees C) is:

$$p_v^* = 0.61094 \exp \left(\frac{17.625T}{T + 243.04} \right)$$

Relative humidity (RH) can be calculated as the ratio of actual water vapour to saturation water vapour pressure ($RH = p_v / p_{sv}$). Assuming a relative humidity of 60%, follow the below steps to use the linearized Penman model to estimate evaporation from a saturated surface where the surface temperature is $T_0 = 25$ degrees C and the air

temperature is $T_a = 20$ degrees C.

- a. [2] Calculate the saturation vapour pressure for both the surface temperature p_o^* and the air temperature p_a^* .

$$p_o^* = 0.61094 e^{\left(\frac{17.625 T_o}{T_o + 243.04}\right)}$$
$$p_o^* = 0.61094 e^{\left(\frac{17.625 * 25}{25 + 243.04}\right)}$$
$$p_o^* = 3.1617 \text{ kPa}$$

$$p_a^* = 0.61094 e^{\left(\frac{17.625 T_a}{T_a + 243.04}\right)}$$
$$p_a^* = 0.61094 e^{\left(\frac{17.625 * 20}{20 + 243.04}\right)}$$
$$p_a^* = 2.3334 \text{ kPa}$$

[1] for approach, [0.5] for numerical answer, [0.5] for units. Other units (eg hPa, Pa) are fine as long as number was converted correctly.

- b. [2] Assuming 60% relative humidity, calculate the actual vapour pressure in the air, p_a .

$$RH = \frac{p_a}{p_a^*}$$
$$0.6 = \frac{p_a}{2.3334}$$
$$p_a = 0.6 * 2.3334$$
$$p_a = 1.4001 \text{ kPa}$$

- c. [2] The ideal gas law tells us that $pV = \frac{m}{M}RT$, where R is the ideal gas constant (8.31 J/K/mol), m is the mass of water present in a parcel of air and M is the molar mass of water (18.02 grams / mol); p is pressure, V is volume, and T is temperature (all in base SI units, i.e. K for temperature). Rearrange this equation for $\rho_a^* = m/V = \underline{\hspace{2cm}}$, a density in kg/m³.

Note: The molar mass of dry air is 28.96 g/mol, and the molar mass of saturated air is 28.44 g/mol. You don't need that for this question.

$$pV = \frac{m}{M}RT$$

$$\rho_a^* = \frac{m}{V} = \frac{p_a M}{RT}$$

- d. [2] Substituting the saturation vapour pressure from (a) into the p term lets you solve for the saturation vapour density. Do this for the surface, and for the air, and calculate the actual vapour density of the air.

$$\rho_a^* = \frac{m}{V} = \frac{p_a M}{RT_a}$$

$$\rho_a^* = \frac{2333 \text{ Pa} * .001802 \text{ kg / mol}}{8.31 \text{ J / K / mol} (20 + 273.15) \text{ K}}$$

$$\rho_a^* = \frac{2333 \text{ Pa} * .001802 \text{ kg / mol}}{8.31 \text{ J / K / mol} (20 + 273.15) \text{ K}}$$

$$\rho_a^* = 0.001036 \frac{\text{kg m}^{-1} \text{s}^{-2} \text{kg mol}^{-1}}{\text{kg m}^2 \text{s}^{-2} \text{K}^{-1} \text{mol}^{-1} \text{K}}$$

$$\rho_a^* = 0.001726 \frac{\text{kg}}{\text{m}^3}$$

OR

$$\rho_a^* = 17.26 \frac{\text{g}}{\text{m}^3}$$

$$\rho_0^* = \frac{3161 \text{ Pa} * .001802 \text{ kg / mol}}{8.31 \text{ J / K / mol} (25 + 273.15) \text{ K}}$$

$$\rho_0^* = 22.99 \text{ g/m}^3$$

$$\rho_a = \frac{1400 \text{ Pa} * .001802 \text{ kg / mol}}{8.31 \text{ J / K / mol} (20 + 273.15) \text{ K}}$$

$$\rho_a = 10.36 \text{ g/m}^3$$

*Note: Students should not use $R_s = 287 \text{ J/kg/K}$ from question 2, as that is the specific gas constant for DRY AIR, not for water, and here we're interested in the partial pressure of water. However, if they took the right approach, and just used the wrong R, give 1.5 out of 2 marks. I didn't really cover this in class.

[1] for right approach

[0.25] for units

[0.25] each for values for ρ_a^* , ρ_0^* , ρ_a

- e. [1] Calculate the vapour density deficit vdd_a .

$$vdd_a = \rho_a^* - \rho_a = 17.26 - 10.36 \text{ g/m}^3 = 6.9 \text{ g/m}^3$$

- f. [1] Calculate the linearized change in saturation vapour density with temperature, s :

$$s = \frac{\Delta \rho_v^*}{\Delta T}$$

$$s = \frac{\Delta \rho_v^*}{\Delta T} = \frac{\rho_a^* - \rho_0^*}{T_a - T_0}$$

$$s = \frac{17.26 - 22.99 \text{ g/m}^3}{20 - 25 \text{ K}}$$

$$s = 1.146 \text{ g/m}^3/\text{K}$$

$$s = .001146 \text{ kg/m}^3/\text{K}$$

[1] for approach

[.5] for value (if they got the wrong values above that'll show up again here, but I haven't been making the value)

[.5] for units (either in g or kg as long as conversion is right)

- g. [2] Assume an aerodynamic resistance for heat of $r_{aH} = 100 \text{ s/m}$. Assume the ground heat flux is 20 W/m^2 . Assume net radiation is 400 W/m^2 . Estimate the latent heat flux.

I didn't go into enough details about the psychrometric constant in class for this to be a fair question, so have dropped this part of the question from the assignment!!!

$$Q_E = \frac{s}{s + \gamma} (Q^* - Q_G) + \frac{C_a \times vdd_a / r_{aH}}{s + \gamma}$$

For next year: don't use partial vapour density, that just makes life harder ☹ .
And spend longer in class talking about the psychrometric constant and its units and different subfields' different definitions.

8. [10]

Identify a microclimate (you may find one on/off campus, or use something like GoogleEarth). Make sure it has a distinctly different microclimate nearby (e.g. opposite sides of a hill, or opposite sides of a tree), to allow for easy comparison between two locations for the following questions.

a. [1] Provide an image your microclimate.

b. [5] Speculate on how each term of the surface energy budget in your microclimate is modulated by the physical surface properties of the microclimate.

[1] each for:

- latent heat
- sensible heat
- something about shortwave radiation
- something about longwave radiation

(4 points)

1 additional point for one of:

- ground heat uptake/release
- breaking down incoming/absorbed/emitted shortwave and/or longwave radiation

c. [4] Discuss the aerodynamic properties of your microclimate. Use terms and concepts from class to discuss how physical attributes of your microclimate interact with flow in the atmosphere (2 points per attribute paired with a discussion of the physics).

For example:

- i. if you choose a hill, talk about how tall the hill is and how flow moves around your hill based on wind speed – relate to the Froude number.
- ii. Or, if you choose a grass/forest transition, talk about the aerodynamic roughness and discuss how wind speeds and momentum production change.
- iii. Or, talk about the Richardson number and how vertical motions over your microclimate might differ from neighbouring regions.

This is not an exhaustive list.

Rubric:

a – [1] for photo

b – [1] per term of the surface energy budget

c – [2] per topic + relation to physical attributes of the system.